

3. [10 points] A group of students planted a pine tree and an oak tree alongside the Diag. Let $P(t)$ and $O(t)$ be the height (in feet) of the pine and the oak t years after they were planted, where

$$P(t) = 170 - 165A^{-0.02t} \quad \text{and} \quad O(t) = \frac{140}{2 + 100e^{-0.3t}}$$

where $A > 1$ is a constant. *For this problem, your answers should be in exact form or accurate up to the first two decimal places.*

- a. [2 points] How tall (in feet) were each of the trees when they were planted?

Answer: Pine: 5

Oak: $\frac{140}{102} \approx 1.372$

- b. [4 points] Ten years after the trees were planted, the height of the pine was 38 ft. Find the value of A . *Find your answer algebraically and show all your work.*

Answer:

$$\begin{aligned} 170 - 165A^{-0.2} &= 38 \\ A^{-0.2} &= \frac{132}{165} = 0.8 \\ A &= (0.8)^{-5} \end{aligned}$$

Answer: $A = \underline{(0.8)^{-5} \approx 3.051}$

- c. [4 points] How many years after being planted does it take the oak to be 38 ft? *Find your answer algebraically and show all your work.*

Answer:

$$\begin{aligned} \frac{140}{2 + 100e^{-0.3t}} &= 38 \\ 140 &= 76 + 3800e^{-0.3t} \\ e^{-0.3t} &= \frac{64}{3800} \\ -0.3t &= \ln\left(\frac{64}{3800}\right) \\ t &= -\frac{1}{0.3} \ln\left(\frac{64}{3800}\right). \end{aligned}$$

Answer: $-\frac{1}{0.3} \ln\left(\frac{64}{3800}\right) \approx 13.612$ years