

4. [10 points] After the students planted the pine and the oak, the university has been monitoring the growth and health of the trees. Fifteen years after being planted, an invasion of cankerworms (a type of caterpillar) is found on the oak. It is predicted that the number of cankerworms (in hundreds) in the oak  $s$  weeks after the pest was detected is given by

$$C(s) = 2e^{0.35s}.$$

- a. [2 points] By what percent is the population of cankerworms expected to grow every week?

**Answer:**

Since  $b = e^{0.35}$ , then  $r = e^{0.35} - 1$ . The population grows by  $100r\% = 100(e^{0.35} - 1)\%$  every week.

**Answer:** The population grows by  $100(e^{0.35} - 1)\% \approx 41.9\%$  every week

- b. [3 points] Let  $F(m)$  be the number of cankerworms in the oak (in thousands)  $m$  days after the pest was detected. Find a formula for  $F(m)$  in terms of  $m$  only.

**Answer:**  $F(m) =$   $0.2e^{0.05m}$

- c. [5 points] A population of weevils (another insect) invades the pine. It is estimated that the population of weevils increases by 44 percent every 2 weeks. How many weeks does it take for the population of weevils to triple? *Show all your work and round your answer to the nearest week.*

**Answer:**

If the population of weevils is given by  $W(t) = ab^t$ , then the fact that it increases by 44 percent every 2 weeks yields  $W(2) = 1.44W(0)$ . In other words

$$\begin{aligned} ab^2 &= 1.44a \\ b &= \sqrt{1.44} = 1.2. \end{aligned}$$

The time  $T$  it takes the population to triple satisfies  $W(T) = 3W(0)$ . Hence

$$\begin{aligned} a(1.2)^T &= 3a & (1.2)^T &= 3. \\ T \ln(1.2) &= \ln(3) & T &= \frac{\ln(3)}{\ln(1.2)} \end{aligned}$$

**Answer:**  $\frac{\ln(3)}{\ln(1.2)} \approx 6$  weeks