

6. [11 points] A company designs chambers whose interior temperature can be controlled. Their chambers come in two models: Model A and Model B.
- a. [5 points] The temperature in Model A goes from its minimum temperature of -3°C to its maximum temperature of 15°C and returning to its minimum temperature three times each day. The temperature of this chamber at 10 am is 15°C . Let $A(t)$ be the temperature (in $^\circ\text{C}$) inside this chamber t hours after midnight. Find a formula for $A(t)$ assuming it is a sinusoidal function.

The amplitude of $A(t)$ is $\frac{15-(-3)}{2} = 9$. The period is 8 hours and the midline is $y = \frac{15+(-3)}{2} = 6$. We know that $A(10) = 15$ a maximum value of $A(t)$. Then

$$A(t) = 9 \cos\left(\frac{2\pi}{8}(t - 10)\right) + 6$$

Answer: $A(t) = \underline{9 \cos\left(\frac{2\pi}{8}(t - 10)\right) + 6}$

- b. [6 points] Let $B(t)$ be the temperature (in $^\circ\text{C}$) inside Model B t hours after midnight, where

$$B(t) = 5 - 3 \cos\left(\frac{3}{7}t + 1\right).$$

Find the two smallest positive values of t at which the temperature in the chamber is 6°C . Your answer must be found algebraically. *Show all your work and give your answers in exact form.*

$$\begin{aligned} 5 - 3 \cos\left(\frac{3}{7}t + 1\right) &= 6 & \cos\left(\frac{3}{7}t + 1\right) &= -\frac{1}{3}. \\ \frac{3}{7}t + 1 &= \cos^{-1}\left(-\frac{1}{3}\right) & \frac{3}{7}t + 1 &= 2\pi - \cos^{-1}\left(-\frac{1}{3}\right) \\ t &= \frac{7}{3}\left(\cos^{-1}\left(-\frac{1}{3}\right) - 1\right) & t &= \frac{7}{3}\left(2\pi - \cos^{-1}\left(-\frac{1}{3}\right) - 1\right) \end{aligned}$$