

2. [14 points] A group of divers recently discovered one of the most submerged caverns in the world. As part of the exploration team, Elena descended into the caverns to take measurements of the temperature and pressure at different depths in the water. Elena started her descent at 8 am and reached the bottom of the caverns at 8:30 am. Let

- $A(t)$ be Elena's depth (in meters) during her descent t minutes after 8 am,
- $B(p)$ be the depth (in meters) at which Elena measures a water pressure of p kPa (kilo-Pascals),
- $C(m)$ be the water temperature (in degrees Celsius) at a depth of m meters.

Assume all these functions are differentiable and invertible.

- a. [4 points] Find mathematical expressions that represent each of the sentences below.
- (i) The temperature of the water in degrees Celsius when its pressure is 118 kPa.

Solution: $C(B(118))$

- (ii) The water pressure, in *Pascals*, 2 meters under the water surface ($1 \text{ kPa} = 1000 \text{ Pascals}$).

Solution: $1000B^{-1}(2)$

- b. [3 points] At 8:02am, Elena started recording all the data that she was measuring. Let $F(x)$ be Elena's depth (in meters) x seconds after she started recording data. Find a formula for $F(x)$ in terms of any of the functions A , B or C .

Solution: $F(x) = A\left(2 + \frac{x}{60}\right)$

- c. [4 points] Complete the following sentence using the fact $C'(30) = -0.8$

Solution: As Elena submerges from 29.7 meters to 30 meters below the water surface, the water's temperature ... **decreases by about 0.24° C .**

- d. [3 points] Circle the one statement below that is best supported by the equation

$$(B^{-1})'(18) = 1.5.$$

- (i) The water pressure increases by 18 kPa for every 1.5 meters that Elena submerges.
- (ii) After Elena has submerged 18 meters, she needs to submerge about 1.5 meters more in order for the water pressure to increase by 1 kiloPascal.
- (iii) Once Elena has submerged 18 meters, the water pressure increases about 0.75 kPa as she submerges half a meter more.
- (iv) Once the water pressure is 18 kPa, to reach a depth where the water pressure is 19 kPa, Elena must submerge about 1.5 meters more.
- (v) Once Elena has submerged 18 meters, she has to submerge an additional 0.1 meter for the water pressure to increase by approximately 1.5 kPa.

Solution: (iii)