

6. [10 points] All problems below are independent of each other.

- a. [3 points] Let $m(x) = (1 + x^2)^{3x-4}$. Circle the limit below that represents $m'(2)$. There is only one correct answer.

Solution:

$$(A) \lim_{h \rightarrow 0} \frac{(1 + x^2)^{3x-4} + h - 25}{h}$$

$$(D) \lim_{h \rightarrow 0} \frac{(1 + (2 + h)^2)^{3h+2} - 25}{h}$$

$$(B) \lim_{h \rightarrow 0} \frac{(1 + h^2)^{3h-4} - 25}{h}$$

$$(E) \lim_{h \rightarrow 0} \frac{(5 + h^2)^{3h+2} - 25}{h}$$

$$(C) \lim_{h \rightarrow 0} \frac{(1 + (2 + h)^2)^{3h-4} - 25}{h}$$

$$(F) \lim_{h \rightarrow 2} \frac{(1 + h^2)^{3h+2} - 25}{h}$$

b. [4 points] Let $p(x)$ be a polynomial satisfying all the following properties:

(i) $p(x) = 0$ only at $x = -2, 0, 3$.

(ii) $\lim_{x \rightarrow -\infty} p(x) = -\infty$ and $\lim_{x \rightarrow \infty} p(x) = -\infty$.

Find one possible formula for $p(x)$. There may be more than one correct answer.

Solution: $p(x) = -x^2(x + 2)(x - 3)$

c. [3 points] Let $h(x)$ be a rational function satisfying all the following properties:

(i) $\lim_{x \rightarrow 2} h(x) = 0$ and h is not defined at $x = 2$.

(ii) $\lim_{x \rightarrow \infty} h(x) = 0$.

Find one possible formula for $h(x)$. There may be more than one correct answer.

Solution: $h(x) = \frac{(x - 2)^2}{(x^2 + 1)(x - 2)}$