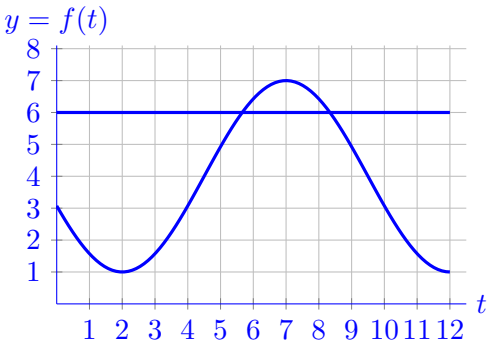
7. [10 points] An apple farmer wants to assess the damage done by a plague to the trees in his orchard. In order to do so, he installs cameras on a couple of small flying robots to film the damage done by the plague to the trees. Let $f(t)$ and $s(t)$ and be the height above the ground (in feet) of the first and second robot t seconds after they started recording.

a. [5 points] Let $f(t) = 4 - 3\cos\left(\frac{\pi}{5}t - \frac{2\pi}{5}\right)$. Find the time(s) at which the first robot is 6 feet above the ground for $0 \le t \le 12$. Your answer(s) should be *exact*. Show all your work.

Solution: From the graph:

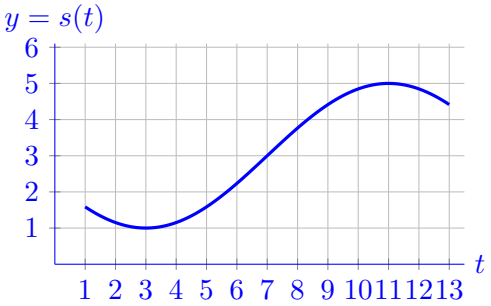


we see that there are two solutions.

$$4 - 3\cos\left(\frac{\pi}{5}t - \frac{2\pi}{5}\right) = 6$$
$$\cos\left(\frac{\pi}{5}t - \frac{2\pi}{5}\right) = -\frac{2}{3}$$
$$\frac{\pi}{5}t - \frac{2\pi}{5} = \cos^{-1}\left(-\frac{2}{3}\right)$$
$$t = \frac{5}{\pi}\left(\cos^{-1}\left(-\frac{2}{3}\right) + \frac{2\pi}{5}\right)$$
$$\frac{\pi}{5}t - \frac{2\pi}{5} = 2\pi - \cos^{-1}\left(-\frac{2}{3}\right)$$
$$t = 2 + \frac{5}{\pi}\left(2\pi - \cos^{-1}\left(-\frac{2}{3}\right)\right)$$
$$t = \frac{5}{\pi}\left(\cos^{-1}\left(-\frac{2}{3}\right) + \frac{2\pi}{5}\right), 2 + \frac{5}{\pi}\left(2\pi - \cos^{-1}\left(-\frac{2}{3}\right)\right)$$

b. [5 points] The graph of the sinusoidal function $s(t)$ is shown below only for $1 \le t \le 13$. Find a formula for $s(t)$.

Solution:



The sinusoidal $s(t) = -A\cos(B(t-h)) + k$ has:

- Amplitude $= 2$ then $A = 2$.
- Midline $y = 3$ then $k = 3$.
- Period $= 16$ then $B = \frac{2\pi}{16} = \frac{\pi}{8}$.
- Horizontal shift $= 3$ then $h = 3$.

Hence $s(t) = -2\cos\left(\frac{\pi}{8}(t-3)\right) + 3$ (other formulas are also possible).

