8. [15 points] Consider the functions $f(x)$ and $g(x)$ given by the formula and graph below.

$$f(x) = \begin{cases} 
2x^3 - 2x^2 & \text{for } x \leq 1, \\
x^3 + 1 & \text{for } x > 1.
\end{cases}$$

- **a.** [5 points] Circle the correct answer(s) in each of the following questions.

  **Solution:**

  i) At which of the following values of $x$ is the function $g(x)$ not continuous?

  \[ x = -3 \quad x = -1 \quad x = 0 \quad x = 2 \quad x = 3.5 \]

  ii) At which of the following values of $x$ is the function $f(x) + g(x)$ continuous?

  \[ x = -2 \quad x = -1 \quad x = 0 \quad x = 1 \quad x = 2 \]

  Note that $g(x)$ is linear on the interval $(-4, -2), (1, 2)$ and $(2, 3)$. All your answers below should be exact. If any of the quantities do not exist, write DNE.

- **b.** [2 points] Find $\lim_{x \to 2}(2f(x) + g(x))$.

  **Solution:** $2(2^3 + 1) - 2.5 = 15.5$

  **Answer:** 15.5

- **c.** [2 points] Find $\lim_{x \to \infty} \frac{f(2x)}{x^3}$.

  **Solution:** $\lim_{x \to \infty} \frac{f(2x)}{x^3} = \lim_{x \to \infty} \frac{8x^3 + 1}{x^3} = 8$

  **Answer:** 8

- **d.** [2 points] Find $\lim_{x \to \infty} g(x^2e^{-x} + 3)$.

  **Solution:** $\lim_{x \to \infty} g(x^2e^{-x} + 3) = \lim_{u \to 3^+} g(u) = 1$

  **Answer:** 1

- **e.** [2 points] For which value(s) of $p$ does $\lim_{x \to p^+} g(x) = 1$?

  **Solution:**

  **Answer:** $p = -3.5, 3$

- **f.** [2 points] Find $\lim_{x \to -1^-} f(-x)$.

  **Solution:**

  **Answer:** 2