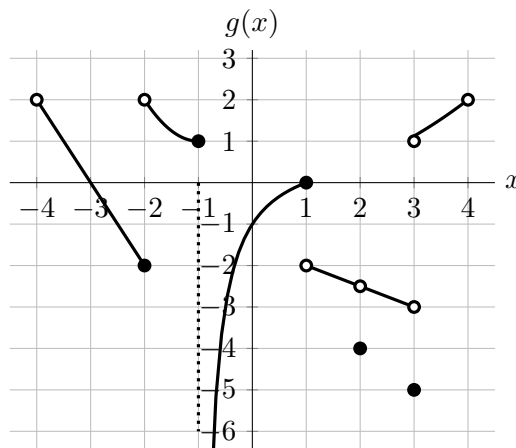


8. [15 points] Consider the functions $f(x)$ and $g(x)$ given by the formula and graph below.

$$f(x) = \begin{cases} 2x^3 - 2x^2 & \text{for } x \leq 1, \\ x^3 + 1 & \text{for } x > 1. \end{cases}$$



- a. [5 points] Circle the correct answer(s) in each of the following questions.

Solution:

- i) At which of the following values of x is the function $g(x)$ not continuous?

$x = -3$ $x = -1$ $x = 0$ $x = 2$ $x = 3.5$

- ii) At which of the following values of x is the function $f(x) + g(x)$ continuous?

$x = -2$ $x = -1$ $x = 0$ $x = 1$ $x = 2$

Note that $g(x)$ is linear on the interval $(-4, -2)$, $(1, 2)$ and $(2, 3)$. All your answers below should be *exact*. If any of the quantities do not exist, write DNE.

- b. [2 points] Find $\lim_{x \rightarrow 2} (2f(x) + g(x))$.

Solution: $2(2^3 + 1) - 2.5 = 15.5$

Answer: 15.5

- c. [2 points] Find $\lim_{x \rightarrow \infty} \frac{f(2x)}{x^3}$.

Solution: $\lim_{x \rightarrow \infty} \frac{f(2x)}{x^3} = \lim_{x \rightarrow \infty} \frac{8x^3 + 1}{x^3} = 8$

Answer: 8

- d. [2 points] Find $\lim_{x \rightarrow \infty} g(x^2 e^{-x} + 3)$.

Solution: $\lim_{x \rightarrow \infty} g(x^2 e^{-x} + 3) = \lim_{u \rightarrow 3^+} g(u) = 1$

Answer: 1

- e. [2 points] For which value(s) of p does $\lim_{x \rightarrow p^+} g(x) = 1$?

Solution:

Answer: $p = -3.5, 3$

- f. [2 points] Find $\lim_{x \rightarrow -1^-} f(-x)$.

Solution:

Answer: 2