

9. [12 points] A new video is released and a few hours later it goes viral. The number of views, in thousands, of the video t hours after it goes viral is given by the function $v(t)$. For the first 24 hours, the number of views of the video is increasing exponentially, reaching 50,000 views 12 hours after going viral and 120,000 views 24 hours after going viral. After that, during the second 24 hours, the video is gaining 10,000 views every 3 hours.
- a. [8 points] Find a piecewise defined formula for $v(t)$ for $0 \leq t \leq 48$. Show all your work.

Solution: For $0 \leq t \leq 24$, let $v(t) = ab^t$, then $ab^{12} = 50$ and $ab^{24} = 120$. Then, dividing, we see

$$\begin{aligned} b^{12} &= 2.4 & b &= (2.4)^{\frac{1}{12}}. \\ ab^{12} &= 50 & \text{yields} & a = \frac{50}{2.4} \end{aligned}$$

For $24 \leq t \leq 48$, $v(t) = a + m(t - 24)$. In this case $v(24) = a = 120$ and $m = \frac{10}{3}$.

$$v(t) = \begin{cases} \frac{50}{2.4}(2.4)^{\frac{t}{12}} & \text{for } 0 \leq t \leq 24 \\ 120 + \frac{10}{3}(t - 24) & \text{for } 24 < t \leq 48. \end{cases}$$

- b. [2 points] Find the hourly percentage growth rate of $v(t)$ during the first 24 hours. Your answer should be given as a percentage accurate up to the first two decimals.

Solution: $r = (2.4)^{\frac{1}{12}} - 1 \approx 0.07568$ then 7.57 %.

- c. [2 points] During the third day, the number of views of the video is not given by a nice formula, but it is at least known that $v'(53) = 6$. What are the units of 6?

Solution: thousands of views per hour.