

1. [13 points] Alex and Misha are running a 100 meter race at Ferry Field. Alex starts the race at the starting line, but because he's on the track team, he lets Misha start the race several meters in front of the starting line.

Five seconds after the race begins, Misha is 32 meters from the starting line. Sixteen seconds into the race, Misha is 92 meters from the starting line, but Alex is crossing the finish line and wins the race.

- a. [2 points] Write a formula for $A(t)$, Alex's distance from the starting line, in meters, assuming that $A(t)$ is a *linear* function.

Solution: We are given that $A(0) = 0$ and $A(16) = 100$, so the vertical intercept is 0 and the slope is $\frac{100-0}{16-0} = \frac{100}{16} = 6.25$.

Answer: $A(t) = \underline{\hspace{10em} \frac{100}{16}t \hspace{10em}}$

- b. [3 points] Write a formula for $M(t)$, Misha's distance from the starting line, in meters, assuming that $M(t)$ is an *exponential* function.

Solution: We are given that $M(5) = 32$ and $M(16) = 92$, and that $M(t) = ab^t$. Then

$$92 = ab^{16}$$

$$32 = ab^5$$

$$\frac{92}{32} = b^{11}$$

$$b = \left(\frac{92}{32}\right)^{\frac{1}{11}} \approx 1.10 \quad \text{and} \quad a = \frac{32}{b^5} = \frac{32}{\left(\frac{92}{32}\right)^{\frac{5}{11}}} \approx 19.8.$$

Answer: $M(t) = \underline{\hspace{10em} 19.8(1.10)^t \hspace{10em}}$

- c. [2 points] How many meters in front of the starting line did Misha start the race?

Answer: $\underline{\hspace{10em} 19.8 \hspace{10em}}$

- d. After their race, Alex and Misha walk north to get some lunch in town. Part way through this trip, Alex realizes he left his watch at the field, and so they turn around and walk south until they return to Ferry Field. They get Alex's watch and then walk north again. Their distance in miles from Ferry Field, t minutes after they leave for the first time, is given by the differentiable function $D(t)$. Some values for $D(t)$ are provided in the table below.

t	3	6	10	12	13	15	19	24
$D(t)$	0.15	0.32	0.57	0.40	0.27	0.03	0.21	0.43

- i. [2 points] What is their average velocity between $t = 10$ and $t = 19$? Include units.

Solution: average velocity = $\frac{0.21 - 0.57}{19 - 10} = -0.04$ miles per min

- ii. [2 points] Estimate their instantaneous velocity 16 minutes into their trip. Include units.

Solution: instantaneous velocity at $t = 16 \approx \frac{0.21 - 0.03}{19 - 15} = 0.045$ miles per min

- iii. [2 points] On which of the following interval(s) **must** $D'(t)$ be less than or equal to zero for **all** values of t in the interval? Circle all correct choices.

[3, 6] [6, 10] [10, 12] [12, 13] [13, 15] NONE OF THESE