3. [8 points] A U of M student is studying the various invasive species of insects she finds in a sample plot of forest.

a. [3 points] On her first visit to the plot, she finds 18 ash borers. She estimates that the number of ash borers will grow by 2.4% each day. Based on this estimate, write a formula for $A(w)$, the number of ash borers in her plot w weeks after her first visit.

Solution: We know that $A(w)$ is of the form ab^w and that $A(0) = a = 18$. We also know that after 1 day, which is $\frac{1}{7}$ of a week, the number of ash borers is 18(1.024). That is,

$$A(1/7) = 18b^{1/7} = 18(1.024)$$

$$b^{1/7} = 1.024$$

$$b = 1.024^7$$

Answer: $A(w) =$ $18(1.024)^{7w}$

b. [5 points] The student's data suggest that, w weeks after her first visit, the number of pineshoot beetles in her plot will be given by

$$P(w) = 11e^{w/6},$$

while the number of gypsy moths will be given by

$$G(w) = 3(1.37)^w.$$

After her first visit, how many weeks will it take for the number of pineshoot beetles to equal the number of gypsy moths? Give your answer in **exact form**.

Solution:

Method 1:

$$11e^{w/6} = 3(1.37)^w$$

$$\ln\left(11e^{w/6}\right) = \ln\left(3(1.37)^w\right)$$

$$\ln(11) + \ln\left(e^{w/6}\right) = \ln(3) + \ln\left(1.37^w\right)$$

$$\ln(11) + w/6 = \ln(3) + w\ln(1.37)$$

$$\ln(11) - \ln(3) = w(\ln(1.37) - 1/6)$$

$$w = \frac{\ln(11) - \ln(3)}{\ln(1.37) - 1/6}$$

Method 2:

$$11e^{w/6} = 2(1.37)^w$$

$$\frac{11}{3} = \frac{1.37^w}{e^{w/6}}$$

$$\frac{11}{3} = \left(\frac{1.37}{e^{1/6}}\right)^w$$

$$\ln\left(\frac{11}{3}\right) = \ln\left(\left(\frac{1.37}{e^{1/6}}\right)^w\right)$$

$$\ln\left(\frac{11}{3}\right) = w\ln\left(\frac{1.37}{e^{1/6}}\right)$$

$$w = \frac{\ln\left(\frac{11}{3}\right)}{\ln\left(\frac{1.37}{e^{1/6}}\right)}$$

Answer: $w = \dfrac{\ln(11) - \ln(3)}{\ln(1.37) - 1/6}$