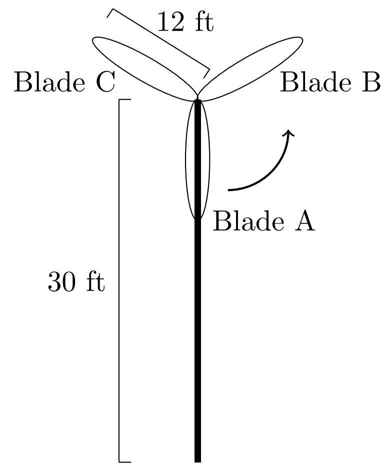
6. [12 points]

a. A wind turbine, spinning counterclockwise at a constant rate, stands 30 feet tall from the ground to its central spinning axis. It has three equally spaced blades, each 12 feet long (as measured from the center of the axis). These blades are labeled as Blade A, B, and C in the figure. At exactly 1:00 pm, an engineer sees that Blade A is pointing straight toward the ground as shown. Blade A then takes exactly 1.5 seconds to return to this downward position. Let $A(t)$ be the height from the ground, in feet, of the outermost tip of Blade A, t seconds after 1:00 pm.

i. [4 points] Write a formula for the trigonometric function $A(t)$.



Solution:

$$A(t) = -12\cos\left(\frac{4\pi}{3}t\right) + 30$$

ii. [3 points] The height $C(t)$ of the outermost tip of Blade C, in feet above the ground, can be given as a transformation of $A(t)$. Circle **all** correct transformations below.

C(t) = A(t - 0.5)	$C(t) = A(t - 2\pi/3)$	$C(t) = A(t) + 18$	$C(t) = A(t - 1)$
$C(t) = A(t + 0.5)$	$C(t) = A(t + 2\pi/3)$	$C(t) = A(t) - 18$	C(t) = A(t + 1)

b. [5 points] The height in feet of the tip of one of the blades on a *different* windmill, t seconds after 1:00pm, is given by

$$W(t) = 24\cos\left(\frac{\pi}{3}t\right) + 60.$$

Find the first two positive times, in seconds, where $W(t) = 40$. Give your answers in **exact form**.

Solution:

$$24\cos\left(\frac{\pi}{3}t\right) + 60 = 40$$

$$\cos\left(\frac{\pi}{3}t\right) = \frac{-20}{24}$$

$$\frac{\pi}{3}t_1 = \cos^{-1}\left(\frac{-20}{24}\right)$$

$$t_1 = \frac{3}{\pi}\cos^{-1}\left(\frac{-20}{24}\right).$$

Now, we use the symmetry of the cosine function. Since the period is 6 and there is no horizontal shift,

$$t_2 = 6 - \frac{3}{\pi}\cos^{-1}\left(\frac{-20}{24}\right)$$

Answer: $t =$ $\frac{3}{\pi}\cos^{-1}(\frac{-20}{24})$ and $t =$ $6 - \frac{3}{\pi}\cos^{-1}(\frac{-20}{24})$

