

9. [8 points] The function $r(x)$ is given by the following formula, where c is a positive constant:

$$r(x) = \begin{cases} \frac{3x+3}{(x+5)(x-2)} & x < 0 \\ \frac{c}{x^3-1} & 0 \leq x < 4 \\ \sqrt{2-\frac{8}{x}} & 4 \leq x. \end{cases}$$

It is not necessary to show work in this problem.

- a. [2 points] Find $\lim_{x \rightarrow -\infty} r(x)$. If the limit does not exist (including the case of limits that diverge to ∞ or $-\infty$), write DNE.

Answer: $\lim_{x \rightarrow -\infty} r(x) = \underline{\hspace{10em} 0 \hspace{10em}}$

- b. [2 points] For what value(s) of x does $r(x)$ have a vertical asymptote? Write NONE if there are no such values.

Answer(s): $x = \underline{\hspace{10em} -5, 1 \hspace{10em}}$

- c. [2 points] For what value(s) of x is $r(x) = 0$? Write NONE if there are no such values.

Answer(s): $x = \underline{\hspace{10em} -1, 4 \hspace{10em}}$

- d. [2 points] For what value(s) of c is the function $r(x)$ continuous at $x = 0$? Write NONE if there are no such values.

Solution: We plug in zero to the first two pieces of $r(x)$ and set those expressions equal to each other:

$$\begin{aligned} \frac{3}{(5)(-2)} &= \frac{c}{-1} \\ c &= \frac{3}{10} \end{aligned}$$

Answer(s): $c = \underline{\hspace{10em} \frac{3}{10} \hspace{10em}}$