2. [5 points] Let 

\[ Q(r) = 1 + r^{\ln(r)}. \]

Use the limit definition of the derivative to write an explicit expression for \( Q'(5) \). Your answer should not involve the letter \( Q \). Do not attempt to evaluate or simplify the limit. Please write your final answer in the answer box provided below.

**Answer:** 

\[
Q'(5) = \lim_{h \to 0} \frac{1 + (5 + h)^{\ln(5 + h)} - (1 + 5^{\ln(5)})}{h}
\]

3. [11 points] Inga, a beekeeper, sets up a new hive on April 1. At two later times, she estimates the hive’s population. These estimates are shown in the table below.

<table>
<thead>
<tr>
<th>weeks after April 1</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>population of the hive, in thousands</td>
<td>7.7</td>
<td>10.9</td>
</tr>
</tbody>
</table>

a. [2 points] Find a formula for a linear function \( L(t) \) modeling the hive’s population, in thousands, \( t \) weeks after April 1.

**Answer:** 

\[ L(t) = \frac{16}{15}(t - 2) + 7.7 \]

b. [4 points] Find a formula for an exponential function \( E(t) \) modeling the hive’s population, in thousands, \( t \) weeks after April 1.

**Solution:** Since \( E(2) = 7.7 \) and \( E(5) = 10.9 \) and we know \( E(t) = ab^t \) for some \( a \) and \( b \),

\[
\begin{align*}
10.9 &= ab^5 \\
7.7 &= ab^2 \\
\frac{10.9}{7.7} &= b^3 \\
b &= \left(\frac{10.9}{7.7}\right)^{1/3} \approx 1.1228 \\
a &= \frac{7.7}{b^2} = \frac{7.7}{\left(\frac{10.9}{7.7}\right)^{2/3}} \approx 6.1076.
\end{align*}
\]

Then \( E(t) = \frac{7.7}{\left(\frac{10.9}{7.7}\right)^{2/3}} \left(\frac{10.9}{7.7}\right)^{t/3} \approx 6.1076(1.1228)^t \).

This problem continues on the next page.
Inga is now studying the populations of two other hives. She determines that the population, in thousands, \( t \) weeks after April 1 of her hive of Carniolan bees can be modeled by

\[
C(t) = 9e^{0.14t},
\]

while the population, in thousands, of her hive of Starline bees can be modeled by

\[
S(t) = 17(1.05)^t.
\]

c. [1 point] By what percent is the hive of Carniolan bees growing each week?

Solution: Since \( e^{0.14} \approx 1.01502 \), we have that the growth rate is \( e^{0.14} - 1 \approx 0.0152 \) or 15.02%.

Answer: \( 15.02 \) %

d. [4 points] At what time \( t \) will the populations of these two hives be equal? Give your answer in exact form, and show every step of your algebraic work.

Solution:

Method 1:

\[
9e^{0.14t} = 17(1.05)^t
\]

\[
\ln(9e^{0.14t}) = \ln(17(1.05)^t)
\]

\[
\ln(9) + \ln(e^{0.14t}) = \ln(17) + \ln(1.05^t)
\]

\[
\ln(9) + 0.14t = \ln(17) + t\ln(1.05)
\]

\[
\ln(9) - \ln(17) = t(\ln(1.05) - 0.14)
\]

\[
t = \frac{\ln(9) - \ln(17)}{\ln(1.05) - 0.14}
\]

Method 2:

\[
9e^{0.14t} = 17(1.05)^t
\]

\[
\frac{9}{17} = \frac{1.05^t}{e^{0.14t}}
\]

\[
\ln\left(\frac{9}{17}\right) = \ln\left(\left(\frac{1.05}{e^{0.14}}\right)^t\right)
\]

\[
\ln\left(\frac{9}{17}\right) = t\ln\left(\frac{1.05}{e^{0.14}}\right)
\]

\[
t = \frac{\ln\left(\frac{9}{17}\right)}{\ln\left(\frac{1.05}{e^{0.14}}\right)}
\]

Answer: \( \frac{\ln(9) - \ln(17)}{\ln(1.05) - 0.14} \)