2. [5 points] Let

$$Q(r) = 1 + r^{\ln(r)}.$$

Use the limit definition of the derivative to write an explicit expression for $Q'(5)$. *Your answer should not involve the letter Q. Do not attempt to evaluate or simplify the limit.* Please write your final answer in the answer box provided below.

Answer: $Q'(5) =$ $$\lim_{h\to 0} \frac{1+(5+h)^{\ln(5+h)} - \left(1+5^{\ln(5)}\right)}{h}$$

3. [11 points] Inga, a beekeeper, sets up a new hive on April 1. At two later times, she estimates the hive's population. These estimates are shown in the table below.

weeks after April 1	2	5
population of the hive, in thousands	7.7	10.9

a. [2 points] Find a formula for a linear function $L(t)$ modeling the hive's population, in thousands, t weeks after April 1.

Answer: $L(t) =$ $\frac{16}{15}(t-2)+7.7$

b. [4 points] Find a formula for an exponential function $E(t)$ modeling the the hive's population, in thousands, t weeks after April 1.

Solution: Since $E(2) = 7.7$ and $E(5) = 10.9$ and we know $E(t) = ab^t$ for some a and b,

$$10.9 = ab^5$$
$$7.7 = ab^2$$
$$\frac{10.9}{7.7} = b^3$$
$$b = \left(\frac{10.9}{7.7}\right)^{1/3} \approx 1.1228 \text{ and } a = \frac{7.7}{b^2} = \frac{7.7}{\left(\frac{10.9}{7.7}\right)^{2/3}} \approx 6.1076.$$

Then $E(t) = \dfrac{7.7}{\left(\frac{10.9}{7.7}\right)^{2/3}} \left(\dfrac{10.9}{7.7}\right)^{t/3} \approx 6.1076(1.1228)^t$.

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Inga is now studying the populations of two other hives. She determines that the population, in thousands, t weeks after April 1 of her hive of Carniolan bees can be modeled by

$$C(t) = 9e^{0.14t},$$

while the population, in thousands, of her hive of Starline bees can be modeled by

$$S(t) = 17(1.05)^t.$$

c. [1 point] By what percent is the hive of Carniolan bees growing each week?

Solution: Since $e^{0.14} \approx 1.01502$, we have that the growth rate is $e^{0.14} - 1 \approx 0.0152$ or 15.02%.

Answer: 15.02 %

d. [4 points] At what time t will the populations of these two hives be equal? Give your answer in **exact form**, and show every step of your algebraic work.

Solution:

Method 1:

$$9e^{0.14t} = 17(1.05)^t$$
$$\ln\left(9e^{0.14t}\right) = \ln\left(17(1.05)^t\right)$$
$$\ln(9) + \ln\left(e^{0.14t}\right) = \ln(17) + \ln\left(1.05^t\right)$$
$$\ln(9) + 0.14t = \ln(17) + t\ln(1.05)$$
$$\ln(9) - \ln(17) = t(\ln(1.05) - 0.14)$$
$$t = \frac{\ln(9) - \ln(17)}{\ln(1.05) - 0.14}$$

Method 2:

$$9e^{0.14t} = 17(1.05)^t$$
$$\frac{9}{17} = \frac{1.05^t}{e^{0.14t}}$$
$$\frac{9}{17} = \left(\frac{1.05}{e^{0.14}}\right)^t$$
$$\ln\left(\frac{9}{17}\right) = \ln\left(\left(\frac{1.05}{e^{0.14}}\right)^t\right)$$
$$\ln\left(\frac{9}{17}\right) = t\ln\left(\frac{1.05}{e^{0.14}}\right)$$
$$t = \frac{\ln\left(\frac{9}{17}\right)}{\ln\left(\frac{1.05}{e^{0.14}}\right)}$$

Answer: $\dfrac{\ln(9) - \ln(17)}{\ln(1.05) - 0.14}$