9. [9 points] You do not need to show work in this problem, but limited partial credit may be awarded for work shown.

a. [5 points] Consider the rational function

$$g(x) = \frac{(Bx^A + 7)(4x - C)}{(3x^2 + 5)(2x - 12)(x - D)},$$

where A, B, C, and D are constants. Suppose that

- $y = 8$ is a horizontal asymptote of $g(x)$
- $x = 5$ is the only vertical asymptote of $g(x)$.

Find the values of A, B, C, and D.

Solution:
For a non-zero horizontal asymptote to exist, the degree of the numerator and denominator must be equal. Since the denominator has degree 4, we must have $A = 3$.

The horizontal asymptote is then equal $\dfrac{4B}{6}$, so for this to equal 8, we must have $B = 12$.

A vertical asymptote at $x = 5$ tells us the denominator must be zero at $x = 5$, so $D = 5$.

Since there is only one vertical asymptote, we know $x = 6$ must be a hole. Thus $4x - C = 0$ when $x = 6$, so $C = 24$.

Answer: $A =$ **3** $B =$ **12** $C =$ **24** $D =$ **5**

b. [4 points] Consider the piecewise function

$$h(x) = \begin{cases} E + \dfrac{28}{3^x + 4} & x \leq 1 \\ \\ G + \dfrac{F}{7^x + 5} & x > 1 \end{cases}$$

where E, F, and G are constants. Suppose that

- $\lim\limits_{x \to \infty} h(x) = 8.5$
- $\lim\limits_{x \to -\infty} h(x) = 12$
- $h(x)$ is continuous at $x = 1$.

Find the values of E, F, and G.

Solution:
As x approaches infinity, $\frac{F}{7^x+5}$ goes to 0, so $G = 8.5$.

As x approaches negative infinity, $\frac{28}{3^x+4}$ goes to $\frac{28}{4} = 7$. Thus $E + 7 = 12$, so $E = 5$.

Since we know $h(x)$ is continuous at 1, we must have

$$E + \frac{28}{3+4} = G + \frac{F}{7^x + 5},$$

which reduces to $9 = 8.5 + \frac{F}{12}$. This gives us $F = 6$.

Answer: $E =$ **5** $F =$ **6** $G =$ **8.5**