5. [10 points] Let us consider the following functions, which concern the productivity of a soybean farm. Bushels are a unit of volume often used to measure a farm's yield.

- Let $Y(b)$ be the yield, in bushels of soybeans, of the farm in the year 2019 when it is infested with b beetles.
- Let $R(s)$ be the revenue, in dollars, of the farm in the year 2019 when it yields s bushels of soybeans.

The functions $Y(b)$ and $R(s)$ are differentiable and invertible.

a. [2 points] Use a complete sentence to give a practical interpretation of the equation

$$R(Y(1{,}200)) = 75{,}000.$$

b. [4 points] Write a single equation representing the following statement in terms of the functions Y, R, and/or their inverses:

> *If there are* 1,600 *beetles, then the farm yields* 200 *bushels of soybeans fewer than are necessary for a revenue of* \$64,000 in the year 2019.

c. [4 points] Complete the following sentence to give a practical interpretation of the equation

$$Y'(1{,}000) = -0.1.$$

If the beetle population was 1,000 *rather than* 950*...*

6. [9 points] A metal bar is unevenly heated, and a laser thermometer is used to measure its temperature at various points. Let $T(q)$ be the temperature of the bar, in degrees Celsius, q feet from its leftmost end. Some values of $T(q)$ are shown in the table below.

q	1	2	3	4	5	6	7	8	9
$T(q)$	40	70	90	80	60	90	130	100	60

a. [3 points] For which of the following intervals of q-values might the function $T'(q)$ be positive for the entire interval? Give your answer as a list of one or more intervals, or write NONE.

$(1, 3)$ $(4, 6)$ $(5, 7)$ $(7, 9)$

b. [3 points] For which of the following intervals of x-values might the function $T(q)$ be concave up for the entire interval? Give your answer as a list of one or more intervals, or write NONE.

$(1, 3)$ $(4, 6)$ $(5, 7)$ $(7, 9)$

c. [3 points] What is the average rate of change of $T(q)$ on the interval $2 \leq q \leq 7$? Include units in your answer.