5. [10 points] Let us consider the following functions, which concern the productivity of a soybean farm. Bushels are a unit of volume often used to measure a farm’s yield.

- Let $Y(b)$ be the yield, in bushels of soybeans, of the farm in the year 2019 when it is infested with $b$ beetles.
- Let $R(s)$ be the revenue, in dollars, of the farm in the year 2019 when it yields $s$ bushels of soybeans.

The functions $Y(b)$ and $R(s)$ are differentiable and invertible.

a. [2 points] Use a complete sentence to give a practical interpretation of the equation

$$R(Y(1,200)) = 75,000.$$ 

b. [4 points] Write a single equation representing the following statement in terms of the functions $Y, R,$ and/or their inverses:

*If there are 1,600 beetles, then the farm yields 200 bushels of soybeans fewer than are necessary for a revenue of $64,000 in the year 2019.*

c. [4 points] Complete the following sentence to give a practical interpretation of the equation

$$Y'(1,000) = -0.1.$$ 

*If the beetle population was 1,000 rather than 950.*

6. [9 points] A metal bar is unevenly heated, and a laser thermometer is used to measure its temperature at various points. Let $T(q)$ be the temperature of the bar, in degrees Celsius, $q$ feet from its leftmost end. Some values of $T(q)$ are shown in the table below.

<table>
<thead>
<tr>
<th>$q$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(q)$</td>
<td>40</td>
<td>70</td>
<td>90</td>
<td>80</td>
<td>60</td>
<td>90</td>
<td>130</td>
<td>100</td>
<td>60</td>
</tr>
</tbody>
</table>

a. [3 points] For which of the following intervals of $q$-values might the function $T'(q)$ be positive for the entire interval? Give your answer as a list of one or more intervals, or write NONE.

$(1, 3)$ $(4, 6)$ $(5, 7)$ $(7, 9)$

b. [3 points] For which of the following intervals of $x$-values might the function $T(q)$ be concave up for the entire interval? Give your answer as a list of one or more intervals, or write NONE.

$(1, 3)$ $(4, 6)$ $(5, 7)$ $(7, 9)$

c. [3 points] What is the average rate of change of $T(q)$ on the interval $2 \leq q \leq 7$? Include units in your answer.