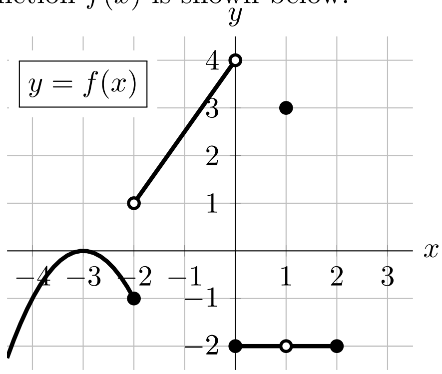
10. [14 points] The graph of the function $f(x)$ is shown below.
(Reduced scale for solutions)



For **a.–b.**, give your answers as a list of one or more of the given numbers, or write NONE

a. [2 points] For which of the values $c = -3, -2, -1, 0, 1$ is $f(x)$ continuous at $x = c$?

Solution: Among these options, $f(x)$ is only continuous when $x = -3$ and when $x = -1$.

b. [2 points] For which of the values $c = -3, -2, -1, 0, 1$ is $\lim\limits_{x\to c^-} f(x) = f(c)$?

Solution: This condition is certainly satisfied at the points where $f(x)$ is continuous, so we already know that $x = -3$ and $x = -1$ should be in our answer. Checking the other points: $\lim\limits_{x\to -2^-} f(x) = -1$ and $f(-2) = -1$. $\lim\limits_{x\to 0^-} f(x) = 4$ but $f(0) = -2$. $\lim\limits_{x\to 1^-} f(x) = -2$ but $f(1) = 3$.
So, our final answer is that the desired condition is satisfied at $\boxed{x = -3, \ -2, \text{ and } \ -1}$.

For **c.–g.**, use the graph of the function $f(x)$ to evaluate each of the expressions below. If a limit diverges to ∞ or $-\infty$ or if the limit does not exist for any other reason, write "DNE." If there is not enough information to evaluate the expression, write "Not enough information."

c. [2 points] $\lim\limits_{x\to 0} f(x)$

Solution: Observe that $\lim_{x\to 0^-} f(x) = 4$ and $\lim_{x\to 0^+} f(x) = -2$. Since the left- and right-hand limits are different, we see that $\lim_{x\to 0} f(x)$ does not exist, so we write "DNE."

d. [2 points] $\lim\limits_{x\to 1} f(x)$

Solution: On either side of $x = 1$, the function $f(x)$ is constant and equal to -2. Therefore $\lim\limits_{x\to 1} f(x) = \boxed{-2}$.

e. [2 points] $\lim\limits_{h\to 0} \dfrac{f(-1+h) - f(-1)}{h}$

Solution: We recognize this expression as $f'(-1)$. Since the function $f(x)$ is linear on the interval $(-2, 0)$, we see that $f'(-1)$ is the slope of this line. We calculate the slope to be $(4-1)/(0-(-2)) = 3/2$.

f. [2 points] $\lim\limits_{x\to 3^+} 4f(x-5) - 1$

Solution: As x approaches 3 from the right, $x - 5$ approaches 2 from the right. This means that $f(x-5)$ approaches 1 from above, and therefore $4f(x-5) - 1$ approaches $4(1) - 1 = \boxed{3}$.

g. [2 points] $\lim\limits_{x\to -3} f(f(x))$

Solution: As x approaches -3 from either side, the value $y = f(x)$ approaches 0 *from negative values only*. Therefore $\lim\limits_{x\to -3} f(f(x)) = \lim\limits_{y\to 0^-} f(y) = \boxed{4}$.