

For **a.-b.**, give your answers as a list of one or more of the given numbers, or write NONE

a. [2 points] For which of the values c = -3, -2, -1, 0, 1 is f(x) continuous at x = c?

Solution: Among these options, f(x) is only continuous when x = -3 and when x = -1.

b. [2 points] For which of the values c = -3, -2, -1, 0, 1 is $\lim_{x \to c^-} f(x) = f(c)$?

Solution: This condition is certainly satisfied at the points where f(x) is continuous, so we already know that x = -3 and x = -1 should be in our answer. Checking the other points: $\lim_{x \to -2^-} f(x) = -1$ and f(-2) = -1. $\lim_{x \to 0^-} f(x) = 4$ but f(0) = -2. $\lim_{x \to 1^-} f(x) = -2$ but f(1) = 3. So, our final answer is that the desired condition is satisfied at x = -3, -2, and -1.

For c.-g., use the graph of the function f(x) to evaluate each of the expressions below. If a limit diverges to ∞ or $-\infty$ or if the limit does not exist for any other reason, write "DNE." If there is not enough information to evaluate the expression, write "Not enough information."

c. [2 points] $\lim_{x \to 0} f(x)$

Solution: Observe that $\lim_{x\to 0^-} f(x) = 4$ and $\lim_{x\to 0^+} f(x) = -2$. Since the left- and right-hand limits are different, we see that $\lim_{x\to 0} f(x)$ does not exist, so we write "DNE."

d. [2 points] $\lim_{x \to 1} f(x)$

Solution: On either side of x = 1, the function f(x) is constant and equal to -2. Therefore $\lim_{x \to 1} f(x) = \boxed{-2}$.

e. [2 points] $\lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$

Solution: We recognize this expression as f'(-1). Since the function f(x) is linear on the interval (-2,0), we see that f'(-1) is the slope of this line. We calculate the slope to be (4-1)/(0-(-2)) = 3/2.

f. [2 points] $\lim_{x \to 3^+} 4f(x-5) - 1$

Solution: As x approaches 3 from the right, x - 5 approaches 2 from the right. This means that f(x-5) approaches 1 from above, and therefore 4f(x-5) - 1 approaches $4(1) - 1 = \boxed{3}$.

g. [2 points] $\lim_{x \to -3} f(f(x))$

Solution: As x approaches -3 from either side, the value y = f(x) approaches 0 from negative values only. Therefore $\lim_{x \to -3} f(f(x)) = \lim_{y \to 0^-} f(y) = 4$.

Creative Commons BY-NC-SA 4.0 International License