2. [12 points]

a. [6 points] Consider the given table of values for the function $j(u)$.

u	1	2	3
$j(u)$	6		54

i. Supposing that $j(u)$ is a linear function, fill in the missing entry in the table. Show your work.

Solution: Since $j(u)$ is linear, its output increases by a constant amount m every time we increment u by 1. The table tells us that when we increment u twice, the output increases by $m + m = 54 - 6 = 48$. Therefore $m = 24$, and so $j(2) = j(1) + m = 6 + 24 = 30$.

ii. Supposing that $j(u)$ is an exponential function, fill in the missing entry in the table. Show your work.

Solution: Since $j(u)$ is exponential, its output increases by a constant multiplicative factor a every time we increment u by 1. The table tells us that when we increment u twice, the output increases by a factor of $a \cdot a = 54/6 = 9$. Therefore $a = 3$, and so $j(2) = j(1) \cdot a = 6 \cdot 3 = 18$.

b. [6 points] A radioactive substance decays exponentially in such a way that, if you have some amount of it, then after 15.2 days you will only have a third as much of it remaining. If I have 110 grams of this substance today, how long will I have to wait until I only have 3 grams remaining? *Show every step of your work, and give your final answer in exact form.*

Solution: Let $r(t)$ denote the amount of the substance that I will have, in grams, t days after today. We are told that the substance decays exponentially, so there are constants P_0 and a such that $r(t) = P_0 a^t$. With this setup, P_0 is the amount of the substance, in grams, that I start with, so $P_0 = 110$. We are told that after 15.2 days, I will have a third of my substance remaining, which gives us the equation

$$\begin{aligned} 110/3 &= 110 \cdot a^{15.2} \\ 1/3 &= a^{15.2} \\ (1/3)^{1/15.2} &= a. \end{aligned}$$

To calculate the time t at which I will have 3 grams remaining, we solve the equation

$$\begin{aligned} 3 &= P_0 a^t \\ 3 &= 110 \cdot \left((1/3)^{1/15.2}\right)^t \\ 3/110 &= \left((1/3)^{1/15.2}\right)^t \\ \ln(3/110) &= t \ln\left((1/3)^{1/15.2}\right) \\ \frac{\ln(3/110)}{\ln\left((1/3)^{1/15.2}\right)} &= t \\ \frac{15.2 \ln(3/110)}{\ln(1/3)} &= t. \end{aligned}$$