

2. [12 points]

a. [6 points] Consider the given table of values for the function  $j(u)$ .

$u$	1	2	3
$j(u)$	6		54

i. Supposing that  $j(u)$  is a linear function, fill in the missing entry in the table.

Show your work.

*Solution:* Since  $j(u)$  is linear, its output increases by a constant amount  $m$  every time we increment  $u$  by 1. The table tells us that when we increment  $u$  twice, the output increases by  $m + m = 54 - 6 = 48$ . Therefore  $m = 24$ , and so  $j(2) = j(1) + m = 6 + 24 = 30$ .

ii. Supposing that  $j(u)$  is an exponential function, fill in the missing entry in the table.

Show your work.

*Solution:* Since  $j(u)$  is exponential, its output increases by a constant multiplicative factor  $a$  every time we increment  $u$  by 1. The table tells us that when we increment  $u$  twice, the output increases by a factor of  $a \cdot a = 54/6 = 9$ . Therefore  $a = 3$ , and so  $j(2) = j(1) \cdot a = 6 \cdot 3 = 18$ .

b. [6 points] A radioactive substance decays exponentially in such a way that, if you have some amount of it, then after 15.2 days you will only have a third as much of it remaining. If I have 110 grams of this substance today, how long will I have to wait until I only have 3 grams remaining? *Show every step of your work, and give your final answer in exact form.*

*Solution:* Let  $r(t)$  denote the amount of the substance that I will have, in grams,  $t$  days after today. We are told that the substance decays exponentially, so there are constants  $P_0$  and  $a$  such that  $r(t) = P_0 a^t$ . With this setup,  $P_0$  is the amount of the substance, in grams, that I start with, so  $P_0 = 110$ . We are told that after 15.2 days, I will have a third of my substance remaining, which gives us the equation

$$\begin{aligned} 110/3 &= 110 \cdot a^{15.2} \\ 1/3 &= a^{15.2} \\ (1/3)^{1/15.2} &= a. \end{aligned}$$

To calculate the time  $t$  at which I will have 3 grams remaining, we solve the equation

$$\begin{aligned} 3 &= P_0 a^t \\ 3 &= 110 \cdot \left( (1/3)^{1/15.2} \right)^t \\ 3/110 &= \left( (1/3)^{1/15.2} \right)^t \\ \ln(3/110) &= t \ln \left( (1/3)^{1/15.2} \right) \\ \frac{\ln(3/110)}{\ln \left( (1/3)^{1/15.2} \right)} &= t \\ \frac{15.2 \ln(3/110)}{\ln(1/3)} &= t. \end{aligned}$$