4. [12 points] Parts **a.** and **b.** below are unrelated.

a. [6 points] Suppose that the temperature in Staunton, Virginia, in degrees Fahrenheit (°F), can be modeled by a sinusoidal function $S(t)$ where t is the time in months since January 1. Note that, for example, August 1 is seven months after January 1. A formula for $S(t)$ is

$$55 - 21\cos\left(\frac{\pi}{6}t\right),$$

i. Using this model, what is the coldest temperature in Staunton?

Solution: The midline of this sinusoidal function is at an output value of 55, and the amplitude is 21. Therefore the lowest value is $55 - 21 = 34$°F.

ii. Using this model, what is the average temperature over the entire year?

Solution: The midline of this sinusoidal function gives us the average temperature, and so the average temperature is 55°F.

iii. At what time t does the temperature first reach "room temperature" (68°F)? *Give your final answer in exact form.*

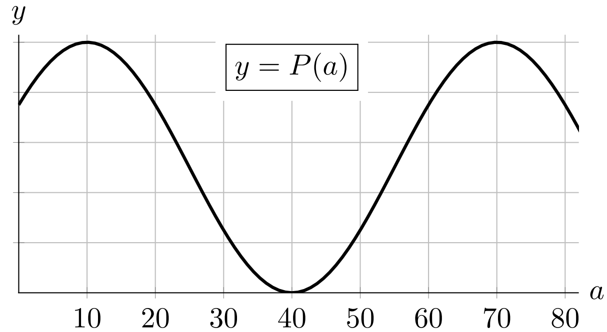
Solution: To find when the temperature first reaches room temperature, we must solve for t in the following equation:

$$55 - 21\cos\left(\frac{\pi}{6}t\right) = 68.$$

Since we are looking for the *first* time the temperature reaches room temperature (and there is no horizontal shift from cosine in the given function), the arccosine function will give us our desired t-value. We therefore solve:

$$\begin{aligned} 55 - 21\cos\left(\frac{\pi}{6}t\right) &= 68 \\ -21\cos\left(\frac{\pi}{6}t\right) &= 68 - 55 \\ \cos\left(\frac{\pi}{6}t\right) &= -13/21 \\ \text{so one solution is given by } \frac{\pi}{6}t &= \arccos\left(-13/21\right) \\ t &= \frac{6}{\pi}\arccos\left(-13/21\right). \end{aligned}$$

b. [6 points] Suppose that a probe lands on some planet other than Earth, and that its recorded temperature, in degrees Fahrenheit, can be modeled by a sinusdoidal function $P(a)$ where a is the time in years since the probe landed. Note that the scale on the y-axis is unknown.



When the temperature is too cold, the probe is in a state of hibernation. The first time it enters hibernation is at $a = 27$.

i. At what time a does the probe leave hibernation?

Solution: The probe will leave hibernation the *next* time after $a = 27$ that $P(a)$ is equal to $P(27)$. Since P has a minimum at $a = 40$, its graph is symmetric about the vertical line $a = 40$, and so the probe will leave hibernation $40 - 27 = 13$ years after $a = 40$. That is to say, the probe will leave hibernation at $a = 53$.

ii. What is the period of $P(a)$?

Solution: We observe that the function P has a maximum at $a = 10$ and another maximum at $a = 70$. Therefore the period of $P(a)$ is $70 - 10 = 60$.

iii. Use the period you found to calculate the next time at which the probe will enter hibernation.

Solution: The probe will next enter hibernation after one period of $P(a)$ has passed since the first time it entered hibernation. Therefore, the probe will next enter hibernation at $a = 27 + 60 = 87$.