5. [10 points] Let us consider the following functions, which concern the productivity of a soybean farm. Bushels are a unit of volume often used to measure a farm's yield.

- Let $Y(b)$ be the yield, in bushels of soybeans, of the farm in the year 2019 when it is infested with b beetles.
- Let $R(s)$ be the revenue, in dollars, of the farm in the year 2019 when it yields s bushels of soybeans.

The functions $Y(b)$ and $R(s)$ are differentiable and invertible.

a. [2 points] Use a complete sentence to give a practical interpretation of the equation

$$R(Y(1{,}200)) = 75{,}000.$$

Solution: This equation tells us that when the farm is infested with 1,200 beetles in 2019, its revenue that year is \$75,000.

b. [4 points] Write a single equation representing the following statement in terms of the functions Y, R, and/or their inverses:

If there are 1,600 *beetles, then the farm yields* 200 *bushels of soybeans fewer than are necessary for a revenue of* \$64,000 in the year 2019.

Solution:

$$Y(1{,}600) = R^{-1}(64{,}000) - 200.$$

c. [4 points] Complete the following sentence to give a practical interpretation of the equation

$$Y'(1{,}000) = -0.1.$$

If the beetle population was 1,000 *rather than* 950. . .

Solution: Since $1{,}000 - 950 = 50$, the equation $Y'(1{,}000) = -0.1$ tells us that $Y(1{,}000)$ is about $0.1 \cdot 50 = 5$ *less* than $Y(950)$. Therefore we may complete the sentence as follows:

If the beetle population was 1,000 *rather than* 950, *the farm would yield about* 5 *fewer bushels of soybeans in* 2019.