6. [9 points] A metal bar is unevenly heated, and a laser thermometer is used to measure its temperature at various points. Let $T(q)$ be the temperature of the bar, in degrees Celsius, q feet from its leftmost end. Some values of $T(q)$ are shown in the table below.

q	1	2	3	4	5	6	7	8	9
$T(q)$	40	70	90	80	60	90	130	100	60

a. [3 points] For which of the following intervals of q-values might the function $T'(q)$ be positive for the entire interval? Give your answer as a list of one or more intervals, or write NONE.

[$(1,3)$] $(4,6)$ [$(5,7)$] $(7,9)$

Solution: The output $T(q)$ increases from $q = 1$ to $q = 2$, and from $q = 2$ to $q = 3$, and so it *is* possible for the derivative $T'(q)$ to always be positive on $(1, 3)$.

The output $T(q)$ decreases from $q = 4$ to $q = 5$, and so it is *not* possible for the derivative $T'(q)$ to always be positive on $(4, 6)$.

The output $T(q)$ increases from $q = 5$ to $q = 6$, and from $q = 6$ to $q = 7$, and so it *is* possible for the derivative $T'(q)$ to always be positive on $(5, 7)$.

The output $T(q)$ decreases from $q = 7$ to $q = 8$, and from $q = 8$ to $q = 9$, and so it is *not* possible for the derivative $T'(q)$ to always be positive on $(7, 9)$.

b. [3 points] For which of the following intervals of q-values might the function $T(q)$ be concave up for the entire interval? Give your answer as a list of one or more intervals, or write NONE.

$(1,3)$ [$(4,6)$] [$(5,7)$] $(7,9)$

Solution: From $q = 1$ to $q = 2$, the function $T(q)$ increases by 30, but from $q = 2$ to $q = 3$, it only increases by 20. If $T(q)$ were concave up on $(1, 3)$, this second increase would have been greater than 30. Therefore $T(q)$ is *not* concave up on $(1, 3)$.

From $q = 4$ to $q = 5$, the function $T(q)$ decreases, and from $q = 5$ to $q = 6$, the function $T(q)$ increases. A concave up function has an increasing first derivative, and these two observations do not preclude an increasing first derivative. Therefore $T(q)$ *might* be concave up on $(4, 6)$.

From $q = 5$ to $q = 6$, the function $T(q)$ increases by 30, and from $q = 6$ to $q = 7$, it increases by 40. A concave up function has an increasing first derivative, and these two observations do not preclude an increasing first derivative. Therefore $T(q)$ *might* be concave up on $(5, 7)$.

From $q = 7$ to $q = 8$, the function $T(q)$ decreases by 30, but from $q = 8$ to $q = 9$, the function $T(q)$ decreases by 40. If $T(q)$ were concave up on $(7, 9)$, this second decrease would have been less than 30. Therefore $T(q)$ is *not* concave up on $(7, 9)$.

c. [3 points] What is the average rate of change of $T(q)$ on the interval $2 \le q \le 7$? Include units in your answer.

Solution: The average rate of change of $T(q)$ on the interval $2 \le q \le 7$ is

$$\frac{T(7) - T(2)}{7 - 2} = \frac{130 - 70}{5} = \frac{60}{5} = 12°\text{C/ft}.$$