7. [9 points] A pizza delivery driver works for a pizzeria on Main Street, which is a long, straight road. The driver tracks her location with her phone while driving a route on Main Street. Let D(t) be her distance from her pizzeria, in miles, at time t hours after noon. Below is a portion of the graph of D'(t), the **derivative** of D(t).



a. [2 points] On which of the following intervals of t is the driver getting closer to her pizzeria for the entire interval? Give your answer as a list of one or more intervals, or write NONE.

(0.1, 0.2) (0.2, 0.3) (0.6, 0.8) (0.8, 1)

Solution: The driver is getting closer to her pizzeria only when D'(t) is negative (i.e. her distance to the pizzeria is *decreasing*). The only one of these intervals on which D'(t) is always negative is (0.6, 0.8).

b. [3 points] The speed limit in the driver's hometown is 40 miles per hour. How many different times does she *begin* to drive over the speed limit?

Solution: She begins to drive over the speed limit at the t-values where |D'(t)| goes from being smaller than 40 to being larger than 40. The t-values at which this happens are:

- right before t = 0.1,
- between t = 0.3 and t = 0.4,
- right before t = 0.7.

Therefore she begins to drive over the speed limit at 3 different times.

- **c**. [2 points] At which of the following times is the driver farthest from her pizzeria? Write the one best answer.
 - t = 0.1 t = 0.35 t = 0.5 t = 0.6 t = 0.7

Solution: Notice that for $0.1 \le t \le 0.7$, the function D'(t) starts out having only non-negative values, and then switches to having only non-positive values. Consider some time t. As long as there are more positive values of D'(t) yet to come, the driver will get farther from the pizzeria than she is now! Of course, once the values of D'(t) are negative, she's now going towards the pizzeria, so her furthest point was in the past. The last point where D'(t) has only ever been non-negative is t = 0.6, and so this is the time at which she is farthest from the pizzeria.

- d. [2 points] Write the number of the the sentence below that best describes the driver's behavior on the interval $0.2 \le t \le 0.5$.
 - 1. The driver keeps returning to the pizzeria to pick up more pizza.
 - 2. The driver is driving on a highway without any traffic.

Solution: Let us consider what a graph for each of these options might look like.

1. For this option, we would expect to see D'(t) going from positive to negative over and over again, since she would be leaving (D'(t) > 0) and then returning to (D'(t) < 0) the pizzeria multiple times.

3.

- 2. For this option, we would expect D'(t) to never be 0, since she wouldn't stop on the highway unless there is traffic.
- 3. For this option, we would expect D'(t) to keep switching from positive (driving forward) to 0 (stopped at a light).
- 4. For this option, we would expect to see D'(t) going from positive to negative over and over again, since she would be driving both towards and away from the pizzeria as she circles around.

Of these descriptions, only number 3 matches the graph, and so this is our answer.

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The driver stops at a series of red lights.