9. [7 points]
We consider formulas for four different rational functions. Let the formulas I-IV be defined as follows:

I. $\dfrac{x^5(x+1)(x-2)}{x+2}$

II. $\dfrac{x^4(x+1)(x-2)(x-8)}{(x-9)(x-3)}$

III. $\dfrac{(x-9)(x+1)}{x-8}$

IV. $\dfrac{(x-3)(x+3)}{x-9}$

We describe three functions. Match each function below with the formula I-IV that could possibly be the formula for that function. Each function below matches with *exactly one* of the formulas above.

A. The function $g(x)$ is such that $\dfrac{g(x)}{x^5}$ diverges to ∞ as $x \to \infty$.

B. The function $h(x)$ is such that the function

$$S(t) = \begin{cases} \sin(2\pi x) & x < 3 \\ h(x) & x \geq 3 \end{cases}$$

is continuous at $x = 3$.

C. The function $f(x)$ is such that $f(x+3)$ has a vertical asymptote at $x = 5$.

Solution: Description A tells us that the numerator of $g(x)$ must have degree at least 6 greater than the degree of its denominator. In Formula I, the difference in degrees between numerator and denominator is $7-1=6$, and so already we see that A matches with I. To double check, we see that the difference in degrees for Formula II is $7-2=5$, for III is $2-1=1$, and for IV is $2-1=1$. Therefore Formula I really is the only possible match for Description A.

Description B tells us that $h(3)$ must be equal to $\lim_{x\to 3^-}\sin(2\pi x)$. But $\sin(2\pi x)$ is always continuous, so this limit is equal to $\sin(2\pi \cdot 3) = 0$. We therefore want a formula that evaluates to 0 at $x = 3$. The numerators of our rational functions are all factored into linear factors, and so this means we must choose the rational function with $(x-3)$ in its numerator. The only option is Formula IV.

Description C tells us that, when we horizontally shift $f(x)$ to the left by 3, there is a vertical asymptote at $x = 5$ for the shifted version of our function. Therefore $f(x)$ itself must have a vertical asymptote at $x = 8$. The denominators of our rational functions are all factored into linear factors, and so this means we must choose the rational function with $(x-8)$ in its denominator. The only option is Formula III.