1. [10 points] Machiko and Nico are learning to sail on a river that runs east-west, and their friend Pennie is walking slowly on a sidewalk parallel to the river, near a bridge across the river. Functions M, N, and P are defined as follows. At the time t minutes after 10:00am,

- $M(t)$ is the position of Machiko's boat,
- $N(t)$ is the position of Nico's boat, and
- $P(t)$ is the position of Pennie.

All three positions are measured in yards east of the bridge.

The table on the right provides some values of these functions. Note that unknown values are shown as "?" in the table.

t (minutes)	1	3	6	7	9
$M(t)$ (yards)	?	24	36	?	?
$N(t)$ (yards)	50	2	?	?	?
$P(t)$ (yards)	7	21	45	40	8

a. [2 points] Find a formula for $M(t)$ assuming that $M(t)$ is a linear function.

Solution: Using the given points $(3, 24)$ and $(6, 36)$, the slope of $M(t)$ is $(36-24)/(6-3) = 12/3 = 4$. Therefore, $M(t) - 24 = 4(t-3)$ or $M(t) = 24 + 4(t-3) = 12 + 4t$.

Answer: $M(t) =$ $24 + 4(t-3) = 12 + 4t$

b. [2 points] Find a formula for $N(t)$ assuming that $N(t)$ is an exponential function.

Solution: Since N is exponential, we have $N(t) = ab^t$, where we can solve for a and b using the given points. We must have $2 = 50b^2$ so $b^2 = \frac{1}{25}$ and hence $b = \frac{1}{5}$. Using the point $(1, 50)$ gives the equation $ab = 50$ so $a = 250$. Therefore, $N(t) = 250(1/5)^t$.

Answer: $N(t) =$ $250(1/5)^t$

c. [2 points] What is Pennie's average velocity between 10:03am and 10:07am? (Include units.)

Solution: $\dfrac{P(7) - P(3)}{7-3} = \dfrac{40-21}{4} = \dfrac{19}{4}$

Answer: $\dfrac{40-21}{7-3} = \dfrac{19}{4} = 4.75$ **yards/minute**

d. [2 points] Estimate $P'(8)$.

Solution: $P'(8) \approx \dfrac{P(9) - P(7)}{9-7} = \dfrac{8-40}{9-7} = \dfrac{-32}{2} = -16$

Answer: $P'(8) \approx$ $\dfrac{8-40}{9-7} = \dfrac{-32}{2} = -16$

e. [2 points] Suppose we also know that Pennie starts at the bridge at 10:00am, walks 50 yards directly east, then turns around and walks directly west back toward the bridge, without changing direction at any other time. For which of the following time intervals could $P'(t)$ be negative for *some* value of t in that interval? Circle all correct choices.

Solution: Note that Pennie must turn around either between 10:03 and 10:07 or between 10:06 and 10:07, but we cannot determine which from the given information. Hence, $P'(t)$ *must* be negative for some value of t in the intervals $[6, 7]$ and $[7, 9]$ and *could* be negative for some value of t in the interval $[3, 6]$.

$[1, 3]$ $\boxed{[3, 6]}$ $\boxed{[6, 7]}$ $\boxed{[7, 9]}$ NONE OF THESE