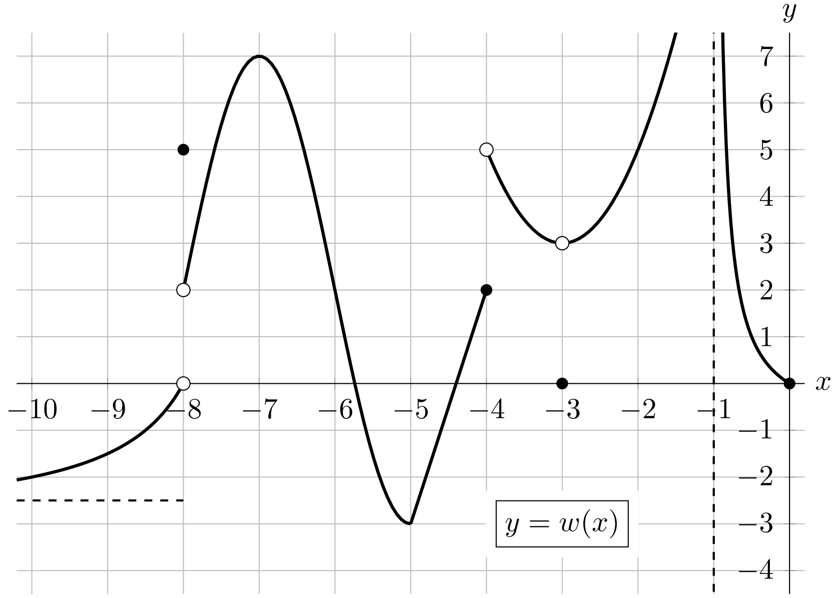
2. [8 points] Below is a portion of a graph of an **even** function $w(x)$. Note that $w(x)$ has a vertical asymptote at $x = -1$, has a horizontal asymptote at $y = -2.5$, and is linear on $[-5, -4]$.



Evaluate each of the given quantities. If the value does not represent a real number (including the case of limits that diverge to ∞ and $-\infty$), write DNE. You do not need to show work in this problem. Give your answers in exact form.

a. [1 point] $\displaystyle\lim_{p\to -4^+} w(p)$

Answer: 5

b. [1 point] $\displaystyle\lim_{x\to -8} w(x)$

Answer: **DNE**

c. [2 points] $\displaystyle\lim_{h\to -1} w(-2+h)$

Solution: For h close to but not equal to -1, the quantity $-2+h$ is close to but not equal to -3. So, as $h \to -1$, we see from the graph that the value of $w(-2+h)$ approaches 3.

Answer: 3

d. [2 points] $\displaystyle\lim_{x\to \infty} w(x)$

Solution: Since w is an even function, the graph of w is symmetric across the vertical axis, so there is a horizontal asymptote on the right at $y = -2.5$. Alternatively, since w is even, we know $w(x) = w(-x)$ for all x in the domain of w, so $\displaystyle\lim_{x\to\infty} w(x) = \lim_{x\to\infty} w(-x) = -2.5$.

Answer: -2.5

e. [2 points] $\displaystyle\lim_{h\to 0}\left((3-h)^2 + \frac{w(-4.5+h) - w(-4.5)}{h}\right)$

Solution:

$$\lim_{h\to 0}\left((3-h)^2 + \frac{w(-4.5+h) - w(-4.5)}{h}\right) = \lim_{h\to 0}\left((3-h)^2\right) + w'(-4.5) = (3-0)^2 + 5 = 14.$$

Answer: 14