5. [8 points] The function $k(x)$ is given by the following formula, where A and B are positive constants:

$$k(x) = \begin{cases} 3 + e^{x-1} & x \leq 1 \\ \dfrac{2x^2+5x+1}{Ax^2+1} & 1 < x < 2 \\ \ln(Bx) + 3 & x \geq 2. \end{cases}$$

a. [2 points] Evaluate each of the expressions below. If a limit does not exist, including if it diverges to ∞ or $-\infty$, write DNE. You do not need to show work.

$\lim\limits_{x\to-\infty} k(x)$

Solution: $\lim\limits_{x\to-\infty} k(x) = \lim\limits_{x\to-\infty}(3 + e^{x-1}) = 3$

Answer: 3

$\lim\limits_{x\to\infty} k(x)$

Solution: $\lim\limits_{x\to\infty} k(x) = \lim\limits_{x\to\infty}(\ln(Bx) + 3) = \infty$ (DNE)

Answer: **DNE**

b. [2 points] Find all horizontal and vertical asymptotes of $k(x)$ or write NONE if there are none.

Solution: By part a., the line $y = 3$ is a horizontal asymptote, and there are no others.

- The first piece of $k(x)$ has no vertical asymptotes since it is a shifted exponential.
- The rational piece of $k(x)$ has denominator that is always positive so there are no vertical asymptotes coming from that piece either.
- The function $\ln(Bx) + 3$ has a vertical asymptote at $x = 0$, but that value of x is not part of the relevant domain for the third piece of $k(x)$.

Hence, $k(x)$ has no vertical asymptotes.

Answer: Horizontal: $y = 3$ Vertical: **NONE**

c. [4 points] Find all values of A and B so that

- $k(x)$ is continuous at $x = 1$ and also
- $k(x)$ is continuous at $x = 2$.

Write NONE if there are no such values. Show your work.

Solution: For continuity at $x = 1$, we consider left and right limits at 1.
Note that $\lim\limits_{x\to1^-} k(x) = 4 = k(1)$ and $\lim\limits_{x\to1^+} k(x) = \frac{8}{A+1}$.
For continuity at $x = 1$ we therefore need $4 = \frac{8}{A+1}$ which gives $A = 1$.
For continuity at $x = 2$, we consider left and right limits at 2.
Note that $\lim\limits_{x\to2^-} k(x) = \frac{8+10+1}{4A+1} = \frac{19}{4A+1}$ and $\lim\limits_{x\to2^+} k(x) = \ln(2B) + 3 = k(2)$.
We found above that $A = 1$, so $\frac{19}{4A+1} = \frac{19}{5} = 3.8$.
For continuity at $x = 2$ we therefore need $\frac{19}{5} = \ln(2B) + 3$, giving $\ln(2B) = \frac{4}{5}$. By definition of the natural logarithm (or exponentiation), we find $2B = e^{4/5}$ and $B = \frac{1}{2}e^{4/5} = 0.5e^{0.8}$.

Answer: $A =$ 1 and $B = \frac{1}{2}e^{4/5} = 0.5e^{0.8}$