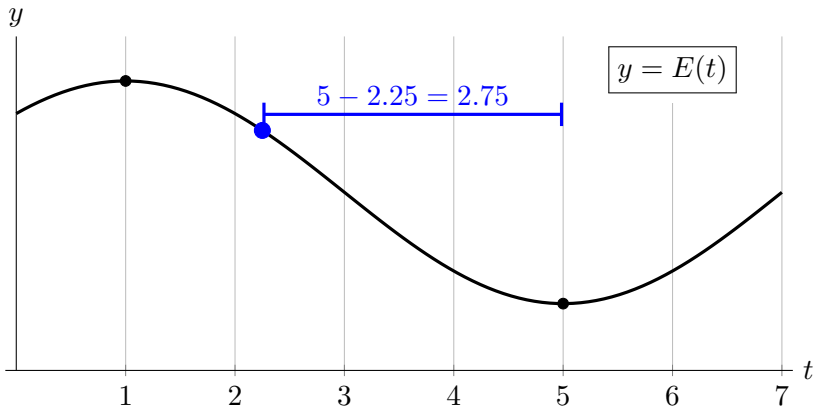
**7**. [7 points]

**a**. [4 points] Zoey, a zoologist, is studying the population of giraffes near a lake. She notices that the number of giraffes near the lake fluctuates in a sinusoidal manner over a 24 hour cycle. The giraffe population reaches a minimum of 30 giraffes at 7:00am every day, and rises to a maximum of 50 giraffes at 7:00pm every day. Let $G(t)$ be a sinusoidal function modeling the number of giraffes at the lake $t$ hours after 6:00am.
Find a formula for $G(t)$.

*Solution:* Because we are given the times for maximum and minimum values, we choose to use transformations of the cosine function to model this situation (though using the sine function requires only a different horizontal shift).
Then $G(t)$ has the form $G(t) = A\cos(B(t-C)) + D$ for some constants $A$, $B$, $C$, and $D$.
The amplitude is $(50-30)/2 = 10$ so $|A| = 10$. The period is 24 so $B = \pi/12$. The midline is at $y = (50+30)/2 = 40$ so $D = 40$. Finally, a minimum occurs at $t = 1$ so we can use a vertical reflection (giving $-\cos$) combined with a horizontal shift to the right by 1. This would give $C = 1$ and $A = -10$. Therefore, one possible equation is $G(t) = -10\cos(\frac{\pi}{12}(t-1)) + 40$.

**Answer:** $G(t) =$ $-10\cos\left(\frac{\pi}{12}(t-1)\right) + 40$

**b**. [3 points] Zoey also studies the population of elephants in the area. Let $E(t)$ be a sinusoidal function modeling the number of elephants at the lake $t$ hours after 6:00am. A portion of the graph of $E(t)$ is shown below.




Give the **exact** values of the next two times $t$ when this model predicts there will be the same number of elephants near the lake as there are at $t = 2.25$ (8:15am). You do not need to show work, but limited partial credit may be awarded for work shown.

*Solution:* To find the first time, we use symmetry of the graph. Because $5 - 2.25 = 2.75$, the next time there will be the same number of elephants near the lake as there are at $t = 2.25$ is at $t = 5 + 2.75 = 7.75$ hours after 6:00 am.
The second time occurs one full period after $t = 2.25$. It takes $5-1 = 4$ hours for the population of elephants to fall from a maximum to a minimum. Therefore, the period of $E(t)$ is 8 and so the second required time is given by $t = 2.25 + 8 = 10.25$ hours after 6:00am.

**Answer:** $t =$ 7.75

**Answer:** $t =$ 10.25