3. [5 points] Suppose $f(t)$ is a differentiable function whose tangent line at the point $t = 1$ is given by the linear function $L(t)$. To the right is a table consisting of some values of $f(t)$ and $L(t)$.

t	-1	1	3	5
$f(t)$	5	2	2	9
$L(t)$	3	2	1	0

a. [1 point] Find the average rate of change of $f(t)$ on the interval $[-1, 5]$.

Solution: $\dfrac{f(5) - f(-1)}{5 - (-1)} = \dfrac{9 - 5}{5 - (-1)} = \dfrac{4}{6} = \dfrac{2}{3}$

Answer: 2/3

b. [1 point] Find the instantaneous rate of change of $f(t)$ at $t = 1$.

Solution: This is equal to the slope of the tangent line $L(t)$ at $t = 1$, which is $\dfrac{L(1) - L(-1)}{1 - (-1)} = \dfrac{2 - 3}{2} = \dfrac{-1}{2}$

Answer: $-1/2$

c. [1 point] Using the table, find the best possible estimate of $f'(-1)$.

Solution: $\dfrac{f(1) - f(-1)}{1 - (-1)} = \dfrac{2 - 5}{1 - (-1)} = \dfrac{-3}{2}$

Answer: $-3/2$

d. [2 points] The function L is invertible, and its inverse function L^{-1} is also linear. Find numbers m and b such that $L^{-1}(x) = mx + b$.

Solution: We have $L(t) = -\frac{1}{2}t + \frac{5}{2}$. So we solve the equation $x = -\frac{1}{2}y + \frac{5}{2}$ for y, which gives $y = -2x + 5$.

Answer: $m =$ -2 and $b =$ 5