6. [10 points] Let $\ell(t) = \dfrac{1}{1+e^{-t}}$.

a. [5 points] Use the limit definition of the derivative to write an explicit expression for $\ell'(-2)$. *Your answer should not involve the name of the function, ℓ. Do not attempt to evaluate or simplify the limit.* Write your final answer in the answer box provided below.

Answer: $\ell'(-2) =$ $\displaystyle\lim_{h\to 0}\frac{\frac{1}{1+e^{-(-2+h)}}-\frac{1}{1+e^{2}}}{h}$ or $\displaystyle\lim_{x\to -2}\frac{\frac{1}{1+e^{-x}}-\frac{1}{1+e^{2}}}{x+2}$

b. [3 points] Evaluate each of the limits below. Give exact answers. If a limit does not exist, including if it diverges to $\pm\infty$, write DNE. You do not need to show work.

i. $\displaystyle\lim_{t\to 0}\ell(t)$

Answer: 1/2

ii. $\displaystyle\lim_{t\to \infty}\ell(t)$

Answer: 1

iii. $\displaystyle\lim_{t\to -\infty}\ell(t)$

Answer: 0

c. [2 points] The function $\ell(t)$ defined above is called the *logistic* function, and you encountered it on Team HW 2. The logistic function is neither even nor odd, but it is possible to shift the graph of $\ell(t)$ to obtain a function that *is* even or odd. Fill in the blanks below to make a TRUE STATEMENT, following these guidelines:

- in the first blank, write UP, DOWN, LEFT, or RIGHT;
- in the second blank, write a positive number;
- in the third blank, write EVEN or ODD.

No justification is needed.

Solution: Any shift (vertical or horizontal) of an even function will have the same limit at $-\infty$ that it has at ∞, so by i. and iii. of part (b) above we know the function we are looking for must be *odd*. And since $\ell(0) = \frac{1}{2}$ and the graph of any odd function must pass through the origin, we need to shift $\ell(t)$ *down* by $\frac{1}{2}$.

"If you shift the graph of $\ell(t)$ DOWN by $\frac{1}{2}$, the resulting function is ODD."