6. [10 points] Let $\ell(t) = \frac{1}{1 + e^{-t}}$.

a. [5 points] Use the limit definition of the derivative to write an explicit expression for $\ell'(-2)$. Your answer should not involve the name of the function, $\ell$. Do not attempt to evaluate or simplify the limit. Write your final answer in the answer box provided below.

\[
\text{Answer: } \ell'(-2) = \lim_{h \to 0} \frac{\frac{1}{1 + e^{-(-2+h)}} - \frac{1}{1 + e^2}}{h} \text{ or } \lim_{x \to -2} \frac{\frac{1}{1 + e^x} - \frac{1}{1 + e^2}}{x + 2}
\]

b. [3 points] Evaluate each of the limits below. Give exact answers. If a limit does not exist, including if it diverges to $\pm \infty$, write DNE. You do not need to show work.

i. $\lim_{t \to 0} \ell(t)$

Answer: \[
\frac{1}{2}
\]

ii. $\lim_{t \to \infty} \ell(t)$

Answer: \[
1
\]

iii. $\lim_{t \to -\infty} \ell(t)$

Answer: \[
0
\]

c. [2 points] The function $\ell(t)$ defined above is called the logistic function, and you encountered it on Team HW 2. The logistic function is neither even nor odd, but it is possible to shift the graph of $\ell(t)$ to obtain a function that is even or odd. Fill in the blanks below to make a TRUE STATEMENT, following these guidelines:

- in the first blank, write UP, DOWN, LEFT, or RIGHT;
- in the second blank, write a positive number;
- in the third blank, write EVEN or ODD.

No justification is needed.

\[
\text{Solution: } \text{Any shift (vertical or horizontal) of an even function will have the same limit at } -\infty \text{ that it has at } \infty, \text{ so by i. and iii. of part (b) above we know the function we are looking for must be odd. And since } \ell(0) = \frac{1}{2} \text{ and the graph of any odd function must pass through the origin, we need to shift } \ell(t) \text{ down by } \frac{1}{2}.\]

“If you shift the graph of $\ell(t)$ \underline{DOWN} by $\frac{1}{2}$, the resulting function is \underline{ODD}.”