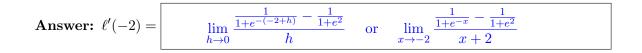
- **6**. [10 points] Let  $\ell(t) = \frac{1}{1 + e^{-t}}$ .
  - a. [5 points] Use the limit definition of the derivative to write an explicit expression for  $\ell'(-2)$ . Your answer should not involve the name of the function,  $\ell$ . Do not attempt to evaluate or simplify the limit. Write your final answer in the answer box provided below.



b. [3 points] Evaluate each of the limits below. Give exact answers. If a limit does not exist, including if it diverges to  $\pm \infty$ , write DNE. You do not need to show work.

i. $\lim_{t \to 0} \ell(t)$		
	Answer:	1/2
ii. $\lim_{t \to \infty} \ell(t)$	Answer:	1
	Allswei	
iii. $\lim_{t \to -\infty} \ell(t)$		
	Answer:	0

- c. [2 points] The function  $\ell(t)$  defined above is called the *logistic* function, and you encountered it on Team HW 2. The logistic function is neither even nor odd, but it is possible to shift the graph of  $\ell(t)$  to obtain a function that *is* even or odd. Fill in the blanks below to make a TRUE STATEMENT, following these guidelines:
  - in the first blank, write UP, DOWN, LEFT, or RIGHT;
  - in the second blank, write a positive number;
  - in the third blank, write EVEN or ODD.

No justification is needed.

Solution: Any shift (vertical or horizontal) of an even function will have the same limit at  $-\infty$  that it has at  $\infty$ , so by i. and iii. of part (b) above we know the function we are looking for must be *odd*. And since  $\ell(0) = \frac{1}{2}$  and the graph of any odd function must pass through the origin, we need to shift  $\ell(t)$  down by  $\frac{1}{2}$ .

"If you shift the graph of  $\ell(t)$  DOWN by  $\frac{1}{2}$ , the resulting function is ODD."