

7. [7 points] Consider the rational function $q(x) = \frac{5x(x+1)(x+3)^2}{(x-2)(x+1)^2(x+3)}$.

a. [2 points] Find all x -values at which the function $q(x)$ has a vertical asymptote.

Answer: $q(x)$ has vertical asymptotes at $x =$ _____

b. [2 points] Find the following limits. If a limit diverges to ∞ or $-\infty$ or does not exist for any other reason, write DNE.

i. $\lim_{x \rightarrow \infty} q(x)$

Answer: _____

ii. $\lim_{x \rightarrow -3} q(x)$

Answer: _____

Suppose the piecewise function $g(x)$ is defined as follows, where $q(x)$ is as above, and k is a constant.

$$g(x) = \begin{cases} e - e^{kx^3} & x \leq 1 \\ q(x) & x > 1 \end{cases}$$

c. [3 points] Find an *exact* value of k for which the function $g(x)$ is continuous at $x = 1$. Show your work.

Answer: $k =$ _____