

2. [8 points] Alana produces a range of kitchenware to honor her favorite comic book writers. Her new “Stan Lee” cups have been especially popular. Let $P(m)$ represent her profit, in thousands of dollars, if she produces m thousand cups, and let $F(m)$ represent the number of followers, in thousands, she will have on social media after she produces m thousand cups. Assume that both functions are invertible and differentiable.

- a. [2 points] Write a complete sentence that gives a practical interpretation of the equation

$$F^{-1}(200) = 8.$$

Solution: Alana has 200,000 followers on social media after producing 8,000 cups.

- b. [2 points] Write a mathematical equation using the functions P , F , and/or their inverses that represents the following statement.

If Alana makes a profit of 30 thousand dollars, she will have 250 thousand followers.

Solution: $F(P^{-1}(30)) = 250$.

- c. [2 points] Complete the following sentence to give a practical interpretation of the equation

$$F'(10) = 32.$$

If Alana produces 12 thousand cups instead of producing 10 thousand cups, ...

Solution: ...she will have about 64,000 more followers.

- d. [2 points] Circle the **one** statement below that is best supported by the equation

$$(P^{-1})'(16) = 4.$$

- i. For every 4 thousand cups Alana produces, she makes an extra 16 thousand dollars in profit, roughly.
- ii. If Alana has made 15.5 thousand dollars in profit, and would like to make 500 more dollars in profit, she will need to produce about 2 thousand more cups.
- iii. If Alana goes from producing 16 thousand cups to producing 17 thousand cups then her profit will increase by about 4 thousand dollars.
- iv. If Alana produces an extra 500 cups after producing her first 4 thousand cups, then she will make about an extra 8 thousand dollars in profit.