

8. [7 points] Gwen and Miles are building sand castles on a beach where the water level varies sinusoidally with the tides over time.

At 9am, when the water level is decreasing, they mark a spot on the beach that is right at the water level at that time. Over the next few hours they build a giant castle on that spot, wondering when the water will return to the base of the castle and start to destroy it. As it turns out, the water reaches the castle again for the first time three hours later, at 12pm.

- a. [3 points] Based on the information above, determine which of the following sinusoidal functions could possibly model the number of meters that the water level is above the base of the sand castle t hours after 9am. *Circle all correct answers.*

i. $\sin\left(\frac{\pi}{3}t\right)$

v. $-\cos\left(\frac{2\pi}{12.5}(t - 1.5)\right) + \cos\left(\frac{2\pi}{12.5}(-1.5)\right)$

ii. $-\sin\left(\frac{\pi}{3}t\right)$

vi. $-\sin\left(\frac{2\pi}{12.5}(t - 1.5)\right) + \sin\left(\frac{2\pi}{12.5}(-1.5)\right)$

iii. $\cos\left(\frac{\pi}{3}t\right)$

iv. $-\cos\left(\frac{\pi}{3}t\right)$

vii. NONE

- b. [2 points] Assuming $f(t)$ is the function which gives the height, in meters, of the water level above the base of the sandcastle t hours after 9am, which function below gives the height in meters of the water above the base of the sand castle m minutes after 10am? *Circle the one correct answer.*

i. $f\left(\frac{m + 60}{60}\right)$

iv. $\frac{f(m - 1)}{60}$

ii. $f(60(m - 1))$

v. $f\left(\frac{m + 1}{60}\right)$

iii. $60f(m + 1)$

vi. NONE

- c. [2 points] The next day, which is a Tuesday, Gwen and Miles return to the spot where they had built their sand castle the day before, and they observe that today the tide reaches that spot at 1pm, while the water level is increasing. Circle all times below when the water level is exactly at the point where they built their sand castle. *Circle all correct answers.*

i. 9am Tue

v. **11am Wed**

ii. **10am Tue**

vi. 1pm Wed

iii. 12pm Tue

vii. NONE

iv. 4pm Tue

Solution: Since there was a three hour gap on Monday between the receding water passing the sand castle and the rising water returning to the same point, this must be true on Tuesday as well, so the water level must have been at the level of the sand castle three hours prior to 1pm, at 10am Tuesday. So the receding water passed the point with the sand castle at 9am Monday and 10am Tuesday, which means the same must happen again 25 hours later at 11am on Wednesday. There are no other times listed when we can know for certain the the water will be at the level of the sand castle.