5. [7 points] Consider the rational functions

$$p(x) = \frac{x(x+3)^3}{7(x-4)^2} \qquad \text{and} \qquad q(x) = \frac{5(x-2)(x-4)}{(x+3)(x+1)(x-2)},$$

and let $R(x) = p(x) \cdot q(x)$ be their product. In (a)–(d) below, circle *all* correct answers or else NONE OF THESE if there are no correct answers.

a. [1 point] Which of the following points belong to the domain of $R(x)$?

$x = -3$ $\quad x = -1$ $\quad x = 0$ $\quad x = 2$ $\quad x = 4$ $\quad$ NONE OF THESE

b. [1 point] At which of the following points does $R(x)$ have a vertical asymptote?

$x = -3$ $\quad x = -1$ $\quad x = 0$ $\quad x = 2$ $\quad x = 4$ $\quad$ NONE OF THESE

c. [1 point] At which of the following points does $R(x)$ have a hole?

$x = -3$ $\quad x = -1$ $\quad x = 0$ $\quad x = 2$ $\quad x = 4$ $\quad$ NONE OF THESE

d. [1 point] Which of the following are horizontal asymptotes of $R(x)$?

$y = 5$ $\quad y = 7$ $\quad y = \frac{5}{7}$ $\quad y = \frac{7}{5}$ $\quad y = 0$ $\quad$ NONE OF THESE

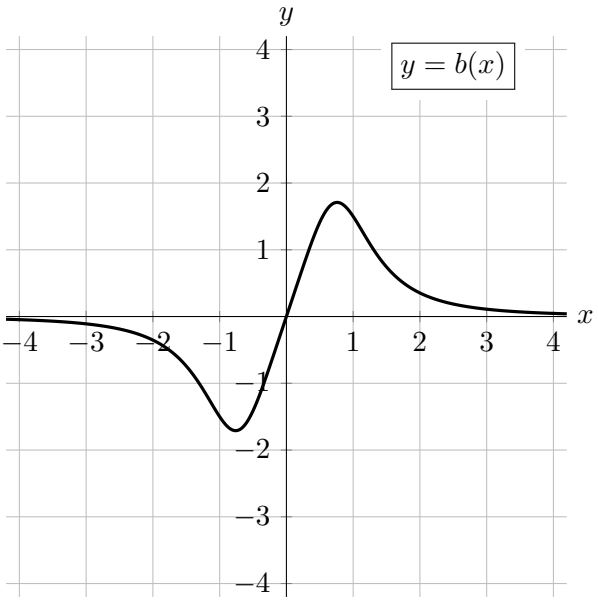
e. [3 points] Compute the following limits, writing DNE if a given limit does not exist.

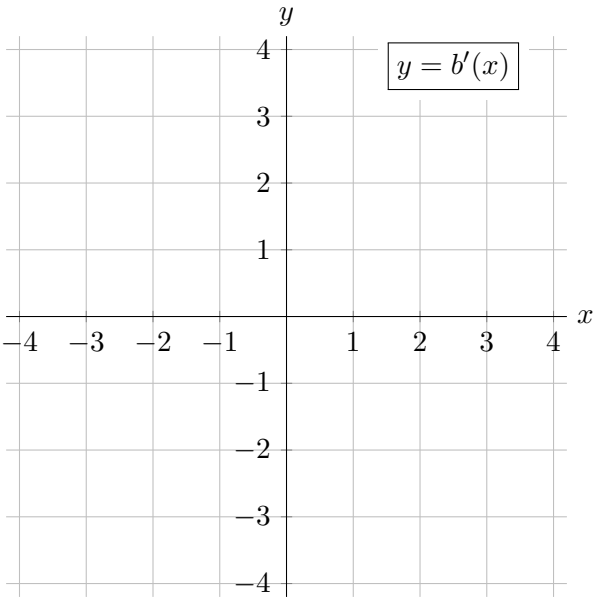
(i) $\lim_{x \to -3} p(x) =$ _____

(ii) $\lim_{x \to -3} q(x) =$ _____

(iii) $\lim_{x \to -3} R(x) =$ _____

6. [6 points] A portion of the graph of the function $b(x)$ is shown below on the left. Carefully sketch the graph of the derivative $b'(x)$ of $b(x)$ for $-4 < x < 4$ on the given axes on the right.





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