1. [11 points] The temperature in degrees Celsius (° C) of a certain cup of hot tea x minutes after it has been poured is given by $T(x) = 20 + 60\left(\frac{1}{2}\right)^{x/30}$.

a. [2 points]

i. What was the initial temperature of the tea in degrees Celsius?

Solution: $T(0) = 20 + 60\left(\frac{1}{2}\right)^0/30 = 20 + 60(1) = 80$.

Answer: 80 ° C

ii. What is $\lim_{x\to\infty} T(x)$?

Solution: Since $\lim_{x\to\infty} (1/2)^{x/30} = 0$, we have $\lim_{x\to\infty} T(x) = 20 + 0 = 20$.

Answer: 20

b. [3 points] The hot tea sits on a table cooling for an entire hour before you remember to drink it. Find the average rate at which the tea cools during this hour. *Your answer should be a positive number. Include units.*

Solution: The average rate of *change* of the temperature of the tea over the first 60 minutes after it was poured is

$$\frac{T(60) - T(0)}{60 - 0} = \frac{20 + \frac{60}{4} - 80}{60} = \frac{15 - 60}{60} = -\frac{45}{60} = -\frac{3}{4} \quad \text{degrees Celsius per minute.}$$

Since we want the average rate of *decrease*, we need to take the absolute value of this number.

Answer: $\frac{3}{4}$° **C per minute, or** 45° **C per hour**

c. [3 points] If t is the temperature of a liquid in degrees Celsius, then its temperature in degrees Fahrenheit is $f(t) = \frac{9}{5}t + 32$.

i. [2 points] Find constants m and b such that $f^{-1}(x) = mx + b$.

Solution: We set $x = \frac{9}{5}t + 32$ and solve for t as a function of x:

$$x - 32 = \frac{9}{5}t, \quad \text{so} \quad f^{-1}(x) = t = \frac{5}{9}(x - 32) = \frac{5}{9}x - \frac{5\cdot 32}{9} = \frac{5}{9}x - \frac{160}{9}$$

Answers: $m =$ 5/9 $b =$ −160/9

ii. [1 point] Write an expression for the temperature of the tea in degrees Fahrenheit x minutes after it has been poured. *Your answer may involve one or both of the letters T or f, but it does not have to; either way, you do not need to simplify.*

Answer: $f\big(T(x)\big)$, **or** $\frac{9}{5}\left(20 + 60\left(\frac{1}{2}\right)^{x/30}\right) + 32$

d. [3 points] Assuming the tea was poured at 12 noon, circle the **one best** practical interpretation of the fact that $(T^{-1})'(50) \approx -1.5$.

i. At 12:50 pm, the tea is cooling at a rate of about 1.5° C per minute.

$\boxed{ii.}$ It takes about three minutes for the tea to cool down from 51° C to 49° C.

iii. 50 minutes after the tea was poured, it takes the tea about 90 seconds to cool down 1° C.

iv. It takes about a minute for the tea to cool down from 51.5° C to 50° C.

v. The tea had a temperature of 60° C fifteen minutes before its temperature was 50° C.