3. [4 points] An insect population in a certain large park varies sinusoidally from a low of 10 million on January 1st to a high of 70 million on July 1st. Let $P(t)$ be the population in the park of this insect, in millions, t months after January 1st. Find a formula for $P(t)$.

Solution: We are looking for a sinusoidal function, of the form $y = A\sin\big(B(t-h)\big) + C$ or $y = A\cos\big(B(t-h)\big) + C$. Since $P(0)$ is a minimum, we will use cosine instead of sine, with A negative, and we do not need any phase shift so $h = 0$. Since $P(t)$ varies between 10 and 70, its amplitude will be $\frac{70-10}{2} = 30$, and the midline will be $\frac{10+70}{2} = 40$. So $A = -30$ and $C = 40$. Finally, since the period is 12 months, we have $B = \frac{2\pi}{12}$. Therefore,

$$P(t) \;=\; -30\cos\left(\frac{2\pi t}{12}\right) + 40.$$

Answer: $P(t) =$ $-30\cos\left(\frac{2\pi t}{12}\right) + 40$

4. [5 points] Let

$$g(x) = \begin{cases} \dfrac{\arctan x}{x} & x \neq 0, \\ 1 & x = 0. \end{cases}$$

You are given that $g'(0)$ exists. Use the limit definition of the derivative to write an explicit expression for $g'(0)$. *Your answer should not involve the letter g. Do not attempt to evaluate or simplify the limit.* Write your final answer in the answer box provided below.

Solution:

$$g'(0) \;=\; \lim_{h\to 0} \frac{g(0+h) - g(0)}{h} \;=\; \lim_{h\to 0} \frac{\frac{\arctan(0+h)}{0+h} - 1}{h} \;=\; \lim_{h\to 0} \frac{\frac{\arctan(h)}{h} - 1}{h}.$$

Answer: $g'(0) =$ $\boxed{\lim_{h\to 0} \frac{\frac{\arctan(h)}{h} - 1}{h}}$