5. [7 points] Consider the rational functions

$$p(x) = \frac{x(x+3)^3}{7(x-4)^2}$$
 and $q(x) = \frac{5(x-2)(x-4)}{(x+3)(x+1)(x-2)}$

and let $R(x) = p(x) \cdot q(x)$ be their product. In (a)–(d) below, circle all correct answers or else NONE OF THESE if there are no correct answers.

a. [1 point] Which of the following points belong to the domain of R(x)?

x = -3 x = -1 x = 0 x = 2 x = 4 None of these

b. [1 point] At which of the following points does R(x) have a vertical asymptote?

$$x = -3$$
 $x = -1$ $x = 0$ $x = 2$ $x = 4$ None of these

c. [1 point] At which of the following points does R(x) have a hole?

$$x = -3$$
 $x = -1$ $x = 0$ $x = 2$ $x = 4$ None of these

d. [1 point] Which of the following are horizontal asymptotes of R(x)?

$$y = 5$$
 $y = 7$ $y = \frac{5}{7}$ $y = \frac{7}{5}$ $y = 0$ NONE OF THESE

e. [3 points] Compute the following limits, writing DNE if a given limit does not exist.

(i)
$$\lim_{x \to -3} p(x) = \underline{0}$$

(ii) $\lim_{x \to -3} q(x) = \underline{DNE}$
(iii) $\lim_{x \to -3} R(x) = \underline{0}$

6. [6 points] A portion of the graph of the function b(x) is shown below on the left. Carefully sketch the graph of the <u>derivative</u> b'(x) of b(x) for -4 < x < 4 on the given axes on the right.



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