8. [8 points] Sound intensity, or loudness, as measured in decibels, decreases as you move further away from a sound source. If you move from a distance of a meters away from a certain sound source to a new distance of b meters away from it, then the sound intensity changes by

$$20\log\left(\frac{a}{b}\right) \text{ decibels.}$$

a. [3 points] Suppose a certain sound source has an intensity of 97 decibels when you stand 9 meters away from it. How far away from the sound source do you need to stand in order to reduce its sound intensity to 90 decibels? *Show all your work, and give your answer in exact form.*

Solution: We are initially standing $a = 9$ meters from the sound souce, and need to determine a new distance b meters away from the sound source so that the loudness changes by $90 - 97 = -7$ decibels. So we need to solve the equation

$$20\log\left(\frac{9}{b}\right) = -7$$

for b. Solving this gives us:

$$\log\left(\frac{9}{b}\right) = -\frac{7}{20}, \qquad \frac{9}{b} = 10^{-7/20}, \qquad b = \frac{9}{10^{-7/20}} = 9 \cdot 10^{7/20}.$$

Answer: $9/(10^{-7/20}) = 9 \cdot 10^{7/20}$ meters

b. [3 points] Write a sentence that gives a practical interpretation, in the context of sound intensity, of the following statement:

$$\text{for any positive number } d, \quad 20\log\left(\frac{d}{10d}\right) = -20.$$

Your sentence should *not* include the variable d.

Solution: Moving 10 times as far away from a sound source decreases its loudness by 20 decibels.

c. [2 points] Suppose the sound intensity S in decibels at a game in the Big House is given as a function of crowd attendance by $S = f(p)$, where p is the number of fans in attendance.

Complete the sentence below to give a practical interpretation of the equation

$$f'(90{,}000) = 10^{-4}.$$

When there are 90,000 *fans in the Big House, you have to add about* 10,000 *more fans in order to increase crowd noise by 1 decibel.*