1. [8 points] Katie is biking on a long, straight path. Her distance, in kilometers (km), from the start of the path t minutes after she begins is given by the function $K(t)$. Some values of $K'(t)$, the **derivative** of $K(t)$, are given in the table below.

t	0	5	10	15	20
$K'(t)$	0	0.27	0.37	0.45	0.5

a. [3 points] Estimate $K''(8)$. Show your work and **include units**.

Solution: We approximate $K''(8)$, the derivative of K' at $t = 8$, by computing the average rate of change of $K'(t)$ over the interval $5 \le t \le 10$, since this is the smallest interval in the table that includes $t = 8$.

$$K''(8) \approx \frac{K'(10) - K'(5)}{10 - 5} = \frac{0.37 - 0.27}{10 - 5} = \frac{0.10}{5} = 0.02.$$

Since the units for the numerator and denominator are kilometers per minute and minutes, respectively, the units on our estimate are km/min^2.

Answer: $K''(8) \approx$ 0.02 $\mathbf{km/min^2}$

b. [1 point] Which **one** of the following statements best describes the first 20 minutes of Katie's ride? Circle the numeral of your answer.

i. Katie started out slowly, but then began to speed up.

ii. Katie started out fast, but then began to slow down as she got tired.

iii. Katie rode at a steady speed for this part of her ride.

c. [4 points] After 20 minutes, Katie has ridden for 7 km, and still has 2 km to go. Find a formula for $L(t)$, the linear approximation to $K(t)$ at $t = 20$, and use it to estimate the total amount of time Katie's bike ride will take.

Solution: The linear approximation $L(t)$ of the function $K(t)$ at $t = 20$ is

$$L(t) = K(20) + K'(20)(t - 20) = 7 + 0.5(t - 20).$$

To estimate the total amount of time Katie's bike ride will take using $L(t)$, we solve the equation $L(t) = 7 + 2 = 9$ for t:

$$\begin{aligned} 7 + 0.5(t - 20) &= 9 \\ 0.5(t - 20) &= 2 \\ t - 20 &= 4 \\ t &= 24. \end{aligned}$$

$L(t) =$ $7 + \frac{1}{2}(t - 20)$

length of Katie's ride $\approx$ 24 minutes