3. [10 points] The number of years, t, that it takes a quantity of money to double when it earns an annual interest rate of $r\%$ is given by

$$t = f(r) = \frac{\ln(2)}{\ln\left(1+\frac{r}{100}\right)}.$$

a. [3 points] Find a formula for $f^{-1}(t)$; that is, solve the above equation for r in terms of t. *Show your work.*

Solution: First, multiply both sides by $\ln\left(1+\frac{r}{100}\right)$ and divide both sides by t to get

$$\ln\left(1+\frac{r}{100}\right) = \frac{\ln(2)}{t}.$$

Then apply the exponential function with base e to both sides to get

$$1+\frac{r}{100} = e^{(\ln 2)/t} = 2^{1/t}.$$

Finally, subtract 1 from both sides and then multiply by 100 to get

$$r = 100\cdot\left(e^{(\ln 2)/t}-1\right) = 100\cdot\left(2^{1/t}-1\right).$$

Answer: $r = f^{-1}(t) =$ $100\cdot\left(e^{(\ln 2)/t}-1\right)$ **or** $100\cdot\left(2^{1/t}-1\right)$

b. [2 points] Write a single equation that represents the statement below. (Your answer may include the letter f.)

Your money takes twice as long to double when it earns 4% interest as it does when it earns 8.2% interest.

Answer: $f(4) = 2f(8.2)$

c. [2 points] Circle the **one** equation below that best represents the following statement:

Your money takes about three months longer to double when it earns 5.2% *interest than when it earns* 5.3% *interest.*

i. $f'(5.2) = 0.25$

ii. $f'(5.2) = -30$

iii. $f'(5.3) - f'(5.2) = 3$

iv. $\boxed{f'(5.2) = -2.5}$

v. $f'(5.2) = 3$

vi. $f(5.3) - f(5.2) = 0.25$

d. [3 points] Complete the sentence below by writing positive numbers in the blanks and circling either HIGHER or LOWER to give a practical interpretation of the following equation:

$$(f^{-1})'(12) = -0.5.$$

If your money will double in 12 *years at the current interest rate, but you want it to double six months earlier, then you will need to invest at an interest rate that is about* 0.25 *percent* $\boxed{\textit{higher}}$ */ lower than the current one.*