4. [12 points] The **continuous** function $w(t)$ is given by the piecewise formula below. Also given is a partial formula for its derivative $w'(t)$. In case it is helpful, recall that $e \approx 2.7$.

$$w(t) = \begin{cases} 3e^t & t \leq 0 \\ \dfrac{1}{3}t^3 - 2t^2 + 3t + 3 & t > 0 \end{cases} \qquad w'(t) = \begin{cases} 3e^t & t < 0 \\ ?? & t = 0 \\ (t-1)(t-3) & t > 0 \end{cases}$$

For parts **a.**, **c.**, and **d.** below, you must use calculus to find and **justify** your answers. Make sure your final conclusions are clear, and that you show enough evidence to justify those conclusions.

a. [2 points] Find $w'(0)$, or write DNE if it does not exist. Show your work or explain.

Solution: Since $w(t)$ is continuous at 0, $w'(0)$ exists if the left and right limits $\lim\limits_{t\to 0^-} w'(t)$ and $\lim\limits_{t\to 0^+} w'(t)$ are the same, in which case $w'(0)$ is their common value. Since

$$\lim_{t\to 0^-} w'(t) \;=\; \lim_{t\to 0^-} 3e^t \;=\; 3e^0 \;=\; 3$$

and

$$\lim_{t\to 0^+} w'(t) \;=\; \lim_{t\to 0^+} (t-1)(t-3) \;=\; (0-1)(0-3) \;=\; 3,$$

we see that $w'(0)$ *does* exist, and equals 3.

Answer: $w'(0) =$ 3

b. [2 points] Find all critical points of $w(t)$, or write NONE if there are none. (*No justification required.*)

Answer: Critical point(s) at $t =$ $1, 3$

c. [4 points] Find the t-coordinates of all global minimum(s) and global maximum(s) of $w(t)$ **on the interval $[-1, 3]$**. If there are none of a particular type, write NONE.

Solution: We just need to evaluate $w(t)$ at its critical points in $[-1, 3]$, and also at the endpoints $t = -1$ and $t = 3$, and determine the greatest and least values from the resulting list. Since

$$w(-1) = 3e^{-1} = \frac{3}{e}, \qquad w(1) = \frac{13}{3}, \qquad \text{and} \qquad w(3) = 3,$$

with $\frac{3}{e} < 3 < \frac{13}{3}$, w has a global min on $[-1, 3]$ at $t = -1$, and a global max on $[-1, 3]$ at $t = 1$.

Answer: Global min(s) at $t =$ -1

Answer: Global max(es) at $t =$ 1

d. [4 points] Find the t-coordinates of all global minimum(s) and global maximum(s) of $w(t)$ **on the interval $(-\infty, \infty)$**. If there are none of a particular type, write NONE.

Solution: Since $\lim\limits_{t\to\infty} w(t) = \lim\limits_{t\to\infty} \left(\frac{t^3}{3} - 2t^2 + 3t + 3\right) = \infty$, $w(t)$ has no global max on $(-\infty, \infty)$. Since $\lim\limits_{t\to-\infty} w(t) = \lim\limits_{t\to-\infty} 3e^t = 0$ and 0 is less than the value of $w(t)$ at all of its critical points (namely $w(1) = \frac{13}{3}$ and $w(3) = 3$), we see that $w(t)$ has no global min on $(-\infty, \infty)$ either.

Answer: Global min(s) at $t =$ **NONE**

Answer: Global max(es) at $t =$ **NONE**