5. [9 points] A function $u(x)$ and its first and second derivatives are given by the formulas

$$u(x) = 2x^2 - \ln|x|, \qquad u'(x) = \frac{(2x-1)(2x+1)}{x}, \qquad \text{and} \qquad u''(x) = \frac{1}{x^2} + 4.$$

Note that $u(x)$ is defined for all real numbers **except** $x = 0$.

a. [2 points] List the x-coordinates of all critical points of $u(x)$. (*No justification required.*)

Answer: Critical points at $x =$ $-1/2,\ 1/2$

b. [4 points] **Use the first derivative test** to find the x-coordinates of all local minima and local maxima of $u(x)$. If there are none of a particular type, write NONE. Be sure you show enough evidence to support your conclusions.

Hint: Remember that $u(x)$ is undefined at $x = 0$.

Solution: In order to apply the first derivative test to the critical points $x = \pm\frac{1}{2}$ of $u(x)$, we need to dermine the sign of $u'(x)$ on either side of $\pm\frac{1}{2}$. Using sign logic, we make the following table of signs:

	$x < -\frac{1}{2}$	$-\frac{1}{2} < x < 0$	$0 < x < \frac{1}{2}$	$\frac{1}{2} < x$
$2x-1$	$-$	$-$	$-$	$-$
$2x+1$	$-$	$+$	$+$	$+$
x	$-$	$-$	$+$	$+$
$u'(x)$	$\frac{-\cdot-}{-} = -$	$\frac{-\cdot+}{-} = +$	$\frac{-\cdot+}{+} = -$	$\frac{+\cdot+}{+} = +$

Or, using a number line:

$u'(x)$: $\frac{-\cdot-}{-} = -$ $\quad -\frac{1}{2} \quad$ $\frac{-\cdot+}{-} = +$ $\quad 0 \quad$ $\frac{-\cdot+}{+} = -$ $\quad \frac{1}{2} \quad$ $\frac{+\cdot+}{+} = +$

Note that even though $x = 0$ is not a critical point of $u(x)$, we need to include it on our number line since $u'(x)$ changes sign there.

Finally, we interpret the signs: since $u'(x)$ changes from negative to positive at both $-\frac{1}{2}$ and $\frac{1}{2}$, each of these points is a local min of $u(x)$. Since $u(x)$ has no other critical points, it has no local max. (Note that $x = 0$ cannot be a local extremum of $u(x)$, since $u(0)$ is undefined.)

Answer: Local min(s) at $x =$ $-1/2,\ 1/2$ and Local max(es) at $x =$ **NONE**

c. [3 points] Note that $u''(x)$ is positive wherever it is defined.

i. How could you use this fact to verify your answer to part **b.**? Briefly explain.

Solution: We could use the Second Derivative Test: since $u''(x)$ is positive near $\pm 1/2$, the graph of $u(x)$ is concave up near $\pm 1/2$, so the critical points $\pm 1/2$ are local minima of $u(x)$.

ii. What does this fact tell you about the number of inflection points of $u(x)$? Briefly explain.

Solution: It tells us that $u(x)$ has no inflection points, since inflection points of $u(x)$ are points at which $u(x)$ changes concavity, or equivalently where $u''(x)$ changes sign.