

5. (6 points) The function $\Phi(x)$ is approximated near $x = 0$ by the 4th degree Taylor polynomial:

$$\Phi(x) \approx x - x^2 + 7x^3 - \frac{\pi}{24}x^4$$

a) (3 pts) Calculate $\Phi'''(0)$.

We can compute this matching the coefficient given for the Taylor polynomial:
 coefficient of $x^3 = 7 = \frac{\Phi'''(0)}{3!} = \frac{\Phi'''(0)}{6}$

b) (3 pts) Is $\Phi(x)$ concave up, concave down, or neither near $x = 0$? Explain *without* using a graph.

So, $\Phi'''(0) = 42.$

Just as above, we can figure:

$$\text{coefficient of } x^2 = -1 = \frac{\Phi''(0)}{2!},$$

$$\text{or } \Phi''(0) = -2.$$

Therefore, Φ is concave down near $x=0$.