6. (15 points)

a) (6 pts) Find the equation of the tangent line of $f(x) = tan(x)$ at the point $x = \pi/4$.

$$f'(x) = \frac{1}{\cos^2 x}\,; \quad f'(\pi/4) = \frac{1}{(\sqrt{2}/2)^2} = 2$$

Tangent line has equation $y = \tan\left(\frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)$

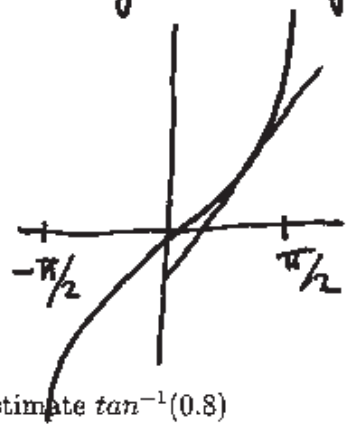
$$y = 1 + 2\left(x - \frac{\pi}{4}\right).$$

b) (2 pts) Use part a) to estimate $tan(3/4)$.

$$\tan\left(\frac{3}{4}\right) \approx 1 + 2\left(\frac{3}{4} - \frac{\pi}{4}\right) = 2.5 - \frac{\pi}{2}.$$

c) (4 pts) Circle **True** or **False**: The actual value of $tan(3/4)$ is greater than your estimation from the previous part. (Briefly comment on the shape of the graph to justify your answer.)

Comment: The graph of $y = \tan x$ looks like this



In particular, it is concave up for $0 < x < \pi/2$. The tangent line lies below the graph for such x.

d) (3 pts) Use part a) to estimate $tan^{-1}(0.8)$

$$x = \tan^{-1}(0.8)$$

implies $\tan x = 0.8$

But $\tan x \approx 1 + 2\left(x - \frac{\pi}{4}\right)$,

so we can solve $0.8 = 1 + 2\left(x - \frac{\pi}{4}\right)$

to get $\boxed{x = \frac{\pi}{4} - 0.1}$ as an estimation of $\tan^{-1}(0.8)$.