7. (12 points) A bungee jumper's height above a river (h in meters) and velocity (v in meters per second — positive v is upward motion) are related. ("Bungee Jumping" is the sport of jumping usually head-first from a tall bridge while securely fastened by an elastic cord. A bungee jumper will bob up and down for some time after being caught by the cord.)

The algebraic relationship between v and h turns out to be:

$$5v^2 + h^2 - 102h = 500$$

a) (3 pts) The jumper later exclaims: "Dude, I was like 36 meters above the river and bouncing up at like 24 meters per second! Rock on!" but his mother suspects he was exaggerating. Demonstrate that his claim is indeed approximately correct.

We want to know v when h is 36: plug $h = 36$ above to get $v^2 = 575.2$. If $v = 24$, $(24)^2 = 576$; since v must be > 0, $v \approx 24$.

b) (6 pts) Using implicit differentiation, calculate $\frac{dv}{dh}$ in terms of v and h.

Differentiate $\quad 5v^2 + h^2 - 102h = 500$

to get

$$10 v \frac{dv}{dh} + 2h - 102 = 0$$

or $\quad \dfrac{dv}{dh} = -2 \dfrac{(h - 51)}{10 v} = \dfrac{51 - h}{5 v}$

c) (3 pts) Calculate $\frac{dv}{dh}$ at the moment described by the jumper in part a).

$$\frac{dv}{dh} = \frac{51 - 36}{5 \cdot \sqrt{575.2}} = \frac{3}{\sqrt{575.2}}$$