8. (18 points) (For full credit, show all your work on each part of this question.) Mushroom growth $G = f(x)$ in a controlled environment is a function of light intensity x. Specifically,

$$G = f(x) = (x^2 + 2x + 2 - Q) * e^{-x},$$

where Q is a constant depending on the species of mushroom. We did some differentiation for you:

$$f'(x) = (-x^2 + Q) * e^{-x}$$
$$f''(x) = (x^2 - 2x - Q) * e^{-x}$$
$$f'''(x) = (-x^2 + 4x - 2 + Q) * e^{-x}$$

a) (5 pts) Taking Q to be an unknown constant, find the values of all critical points of this function, assuming its domain is $(-\infty, \infty)$.

Set $f'(x) = 0 = (-x^2 + Q)\, e^{-x}$.
This implies $0 = -x^2 + Q$, or $x^2 = Q$.
If $Q < 0$, there are no critical points. If $Q \geq 0$, $x = \pm\sqrt{Q}$

b) (2 pts) The Greater Mycoparadeigma mushroom has $Q = .81$. What are the critical points of its growth function? Again assume its domain is $(-\infty, \infty)$.

$$x = \pm\sqrt{.81} = \pm 0.9$$

c) (6 pts) Is each critical point found above a local minimum, a local maximum, or neither? (Use any method, but indicate how you know.)

Check the second derivative f'' at ± 0.9:

$$f''(0.9) = (0.81 - 2\cdot 0.9 - 0.81)\, e^{-0.9} < 0,$$
$$f''(-0.9) = (0.81 + 2\cdot 0.9 - 0.81)\, e^{+0.9} > 0.$$

So, $x = 0.9$ is a local max, $x = -0.9$ local min.

d) (5 pts) Your variable-intensity bulb can be set at x=0, x=4, or anywhere in between. What is the optimal lighting intensity for the Greater Mycoparadeigma? Show your work so we know you have been thorough!

The local max for $0 < x < 4$ is at $x = 0.9$ by part (c). We still need to compare this to the endpoints: $f(0) = 2 - 0.81$
$f(4) = (22 - 0.81)\cdot e^{-4} \leq \frac{22}{16}$. But $f(0.9) \approx 1.54$
≈ 1.38

Or, you can observe that $f'(x) < 0$ for all $x > 0.9$, so f decreases from 0.9 to 4.