

3. (9 points) [Show your work.] Use the information given in the table to find $h'(4)$ if:

x	1	2	3	4
$f(x)$	2	1	4	2
$f'(x)$	3	2	-1	2
$g(x)$	4	2	1	3
$g'(x)$	3	2	2	-3

$$h'(x) = \frac{g'(x)f(x) - g(x)f'(x)}{(f(x))^2} \quad h'(4) = \underline{\underline{-3}}$$

(i) $h(x) = g(x)/f(x)$;

$$h'(4) = \frac{(-3)(2) - 3(2)}{4} = \frac{-12}{4} =$$

(ii) $h(x) = f(\sqrt{x})$;

$$h'(x) = f'(\sqrt{x}) \cdot \frac{1}{2} x^{-1/2} =$$

$$h'(4) = \underline{\underline{1/2}}$$

$$h'(4) = f'(2) \cdot \frac{1}{4} = 2 \left(\frac{1}{4}\right)$$

(iii) $h(x) = \ln(g(x))$;

$$h'(4) = \underline{\underline{-1}}$$

$$h'(x) = \frac{1}{g(x)} \cdot g'(x)$$

$$h'(4) = \frac{1}{3} (-3) = -1$$

4. (9 points) On what interval(s) is the function $f(x) = e^{-x^4}$ increasing and concave down? [Show your work.]

$$f'(x) = -4x^3 e^{-x^4}$$

$$f'(x) > 0 \text{ when } x < 0.$$

$$\begin{aligned} f''(x) &= -12x^2 e^{-x^4} + (-4x^3) e^{-x^4} (-4x^3) \\ &= e^{-x^4} (4x^2) (4x^4 - 3) \end{aligned}$$

$$f''(x) < 0 \text{ when } \begin{aligned} 4x^4 &< 3 \\ x^4 &< \frac{3}{4} \end{aligned} \rightarrow |x| < \sqrt[4]{\frac{3}{4}} \\ \text{or } -\sqrt[4]{\frac{3}{4}} < x < \sqrt[4]{\frac{3}{4}}$$

Thus,

ANSWER: f is increasing and concave down on the interval(s):

$$\underline{\underline{\left(-\sqrt[4]{\frac{3}{4}}, 0\right)}}$$