8. (15 points) A boat is pulled toward a dock by a rope from the bow through a ring on the dock 5 feet above its bow. (See figure) The rope is hauled in at a rate of 2 feet per second.

\[ \frac{dy}{dt} = -2 \text{ ft/sec} \]

In answering the following questions, use complete sentences, show your work and use units.

(a) How fast is the boat approaching the dock when 13 feet of rope are out?

\[ \ell^2 = x^2 + 25 \]

\[ 2 \ell \frac{d\ell}{dt} = 2x \frac{dx}{dt} \quad \Rightarrow \quad \frac{dx}{dt} = \frac{\ell}{x} \frac{d\ell}{dt} \]

Thus, \( \frac{dx}{dt} = \frac{13}{5} \text{ ft/sec} \)

When 13 feet of rope are out, the boat is approaching the dock at 13/5 ft/sec, or 2.6 ft/sec.

(b) At what rate is the angle \( \theta \) changing at that time?

\[ \tan \theta = \frac{x}{5} \]

\[ \frac{1}{\cos^2 \theta} \frac{d\theta}{dt} = \frac{1}{5} \frac{dx}{dt} \]

\[ \cos \theta = \frac{5}{13} \]

\[ \frac{d\theta}{dt} = \frac{1}{5} \cos^2 \theta \left( \frac{dx}{dt} \right) = \frac{1}{5} \left( \frac{5}{13} \right)^2 \frac{13}{6} = \frac{5}{78} \]

The angle is decreasing at the rate of \( \frac{5}{78} \) radian per second.