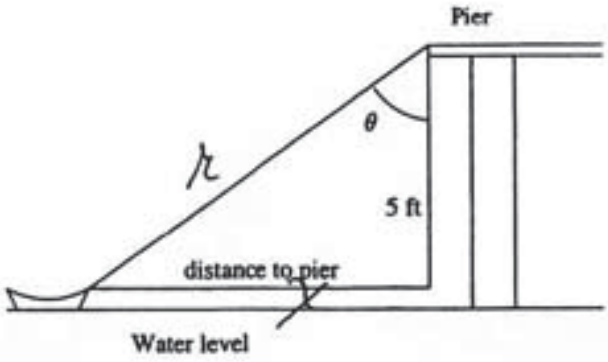
8. (15 points) A boat is pulled toward a dock by a rope from the bow through a ring on the dock 5 feet above its bow. (See figure) The rope is hauled in at a rate of 2 feet per second.



given $\frac{dr}{dt} = -2$ ft/sec

In answering the following questions, use complete sentences, show your work and use units.

(a) How fast is the boat approaching the dock when 13 feet of rope are out?

$r^2 = x^2 + 25$

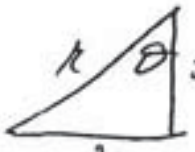
$2r\frac{dr}{dt} = 2x\frac{dx}{dt} \rightarrow \frac{dx}{dt} = \frac{r}{x}\frac{dr}{dt}$

when $r = 13$, $169 - 25 = x^2$, so $x = 12$ (discard -12)

Thus, $\frac{dx}{dt} = \frac{13}{12}(-2) = -\frac{13}{6}$ ft/sec.

When 13 feet of rope are out, the boat is approaching the dock at 13/6 ft per second (or 2 1/6 ft/sec).

(b) At what rate is the angle θ changing at that time?



$\tan\theta = \frac{x}{5}$

$\frac{1}{\cos^2\theta}\frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$

at $s = 13$, $\cos\theta = \frac{5}{13}$

$\frac{d\theta}{dt} = \frac{1}{5}\cos^2\theta\left(\frac{dx}{dt}\right) = \frac{1}{5}\left(\frac{5}{13}\right)^2\left(\frac{-13}{6}\right) = \frac{-5}{78}$

The angle is decreasing at the rate of $5/78 \approx .064$ radians per second.