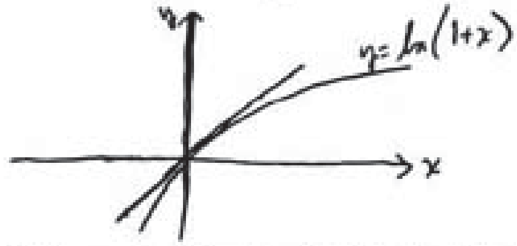
9. (14 points) **(a)** Find the local linearization of the function $f(x) = \ln(1+x)$ near the point $x = 0$. Show your work.

$$f'(x) = \frac{1}{1+x} \qquad f(0) = 0 \quad f'(0) = 1$$

$$f(x) \approx 0 + x = x$$
(near $x = 0$)

(b) Is the approximation to $\ln(1+x)$ given by the local linearization an underestimate or overestimate? Explain why?



The approximation is an overestimate since f is concave down

(c) We saw in Chapter 1 of the text that P_0 dollars invested at a rate of $r\%$ per year grows to be worth $P_0(1 + r/100)^t$ dollars after t years. Compute, in terms of the interest rate r, how long it takes for the invested money to double in value?

$$2P_0 = P_0\left(1+\frac{r}{100}\right)^t \qquad t = \frac{\ln 2}{\ln\left(1+\frac{r}{100}\right)}$$

$$\ln 2 = t \ln\left(1+\frac{r}{100}\right)$$

(d) A common rule of thumb used by investors is the "Rule of 70" — *money invested at a $r\%$ interest per year doubles in value in $70/r$ years.* Explain why this is a reasonable approximation to the actual doubling time.

Since $\ln\left(1+\frac{r}{100}\right) \approx \frac{r}{100}$ (from part (a)), the money doubles in approximately $\frac{\ln 2}{\frac{r}{100}} \approx \frac{.69}{\frac{r}{100}} = \frac{69}{r}$ years, and $\frac{69}{r}$ is close to $\frac{70}{r}$