

(2.) (9 points) Suppose you are given the following data about a differentiable function f :

- $f(3) = 7$
- $f'(3) = -4$.

(a) Find the local linearization of f near $x = 3$.

$$\begin{aligned}f(x) &\approx f(3) + f'(3)(x - 3) \\ &= 7 - 4(x - 3)\end{aligned}$$

(b) Use linear approximation to estimate $f(3.1)$.

$$\begin{aligned}f(3.1) &\approx 7 - 4(3.1 - 3) \\ &= 7 - 4(0.1) = 6.6\end{aligned}$$

(c) If $f''(3) < 0$, do you expect your approximation to be an overestimate or underestimate for $f(3.1)$? Explain, *using a sketch* to support your answer. Include all relevant features of the function on your sketch—and express your answer in a sentence.

We expect our linear approximation to be an overestimate: since the graph is concave down near $x = 3$, the tangent line there is above the graph near $x = 3$.

[Note: A sketch should also be included here.]