(5.) (12 points) Suppose p is a cubic polynomial function. Recall that this means that

$$p(x) = a_3x^3 + a_2x^2 + a_1x + a_0,$$

for some constants a_0, a_1, a_2, a_3, with $a_3 \neq 0$.

(a) If $p(0) = 1$, then what is the value of a_0?

$$a_0 = p(0) = 1$$

(b) If $p'(0) = 1$, then what is the value of a_1?

$$a_1 = p'(0) = 1$$

(c) If $p''(0) = 1$, then what is the value of a_2?

$$p''(0) = 2a_2,$$
$$\text{so } 2a_2 = 1, \text{ and } a_2 = \tfrac{1}{2}$$

(d) If $p'''(0) = 1$, then what is the value of a_3?

$$p'''(0) = 6a_3$$
$$\text{so } 6a_3 = 1, \text{ and } a_3 = \tfrac{1}{6}$$

(e) Find the formula for a cubic polynomial function q that satisfies:

$$q(0) = 2, \quad q'(0) = -1, \quad q''(0) = 5, \quad q'''(0) = 4.$$

[Note: You may use the information that you found in parts (a)-(d) to help you.]

$$q(x) = \tfrac{4}{6}x^3 + \tfrac{5}{2}x^2 - x + 2$$